# Motion Planning in dynamic environments using Model Predictive Control

Maximilian Hartz

University of Hamburg
Faculty of Mathematics, Informatics and Natural Sciences
Department of Informatics
**Technical Aspects of Multimodal Systems**

15. May 2025

# Outline




Motivation

Path vs Motion Planning

CHOMP

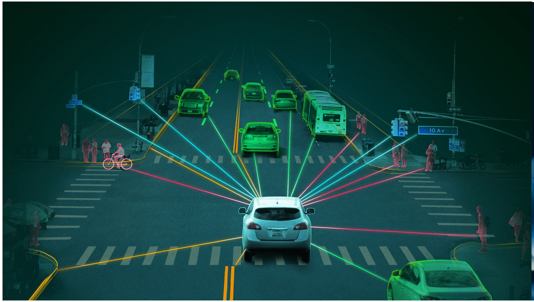
TrajOpt

STOMP

MPC

Comparison

# Motivation




Source: [4]

Source: [2]

# Path Planning vs Motion Planning

## Path Planning

- find shortest path from A to B (globally)
- (near) optimal solution
- long computation
- avoid static but (often) no dynamic obstacles
- no time
- result is a path
  - a series of points

## Motion Planning

- find a smooth path
- follow constrains (speed, angles)
- avoid any obstacle
- incorporate time
- result is a trajectory
  - a series of points at specific times

# Global vs Local

## Global Planning

- may take longer
- optimize trajectory

## Local Planning

- real time
- react to sensor feedback
- deterministic
- needs reference trajectory

# Approaches

- Traditional optimization techniques
  - CHOMP
  - TrajOpt
- Sampling based optimization techniques
  - STOMP
- Global optimization with local planner
  - MPC

# Problem Description

- known start and goal state
- $K$ degrees of freedom
- $T$ time-steps
- optimize trajectory $u \in \mathbb{R}^{K \times T}$
- $u_t$ state at time $t$

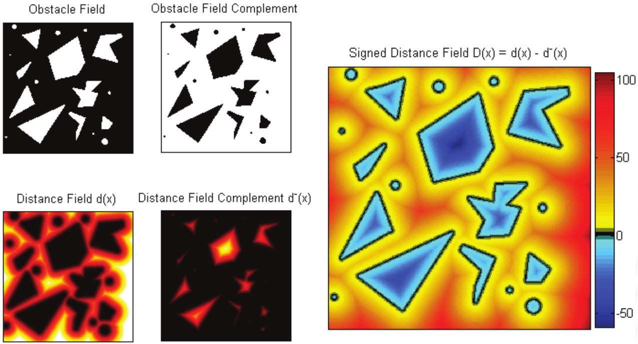
$$\begin{aligned} \min_u \; & L(u) \\ \text{s.t.} \;\; g_i(u) &\leq 0 \\ h_i(u) &= 0 \end{aligned} \tag{1}$$

# CHOMP

- Covariant Hamiltonian Optimization for Motion Planning
- Optimization of cost function
- Gradient Descent
- $L(u) = L_{obs}(u) + L_{prior}(u)$
- obstacle cost $L_{obs}(u)$
- smoothness $L_{prior}(u)$

# CHOMP: obstacle cost





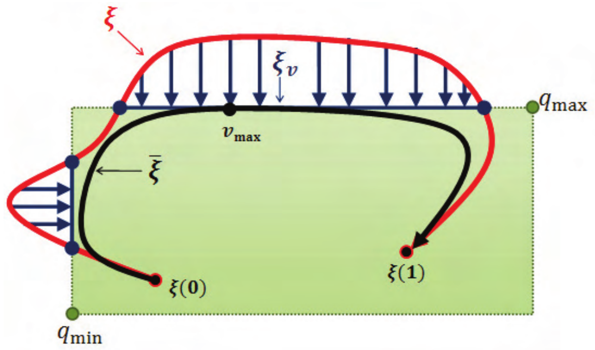

Source: [6]

# CHOMP: smoothness

- acts as regularization
- derivative from finite difference
- any number of derivatives
- $L_{prior}(u) = \frac{1}{2} \sum_{t=0}^{T} \left\| \frac{u_{t+1} - u_t}{\Delta t} \right\|^2$

# CHOMP: Joint Limits






Source: [6]

# TrajOpt

- Sequential Convex Optimization
- solve approximated convex problem
- improve original problem

$$\begin{aligned} \min f(u) \\ \text{s.t.} \quad g_i(u) \leq 0 \\ h_i(u) = 0 \end{aligned}$$

$$\begin{aligned} \min \tilde{f}(u) + \mu \sum \max(\tilde{g}_i(u), 0) + \mu \sum |\tilde{h}_i(u)| \\ \text{s.t.} \quad ||(u - u^*)|| < s \end{aligned}$$

- Trust region size $s$
  - increases if $x^*$ is better
  - decreases until $x^*$ is better
- constrains might be broken
  - check after optimization
  - not satisfied $\Rightarrow$ increase $\mu$

1: **for** PenaltyIteration $= 1, 2, \ldots$ **do**<br>
2:   **for** ConvexifyIteration $= 1, 2, \ldots$ **do**<br>
3:     $\tilde{f}, \tilde{g}, \tilde{h} = \text{ConvexifyProblem}(f, g, h)$<br>
4:     **for** TrustRegionIteration $= 1, 2, \ldots$ **do**<br>
5:       $x \leftarrow \arg\min_{x} \tilde{f}(\mathbf{x}) + \mu \sum_{i=1}^{n_{ineq}} |\tilde{g}_i(\mathbf{x})|^+ + \mu \sum_{i=1}^{n_{eq}} |\tilde{h}_i(\mathbf{x})|$<br>
        subject to trust region and linear constraints<br>
6:       **if** TrueImprove / ModelImprove $> c$ **then**<br>
7:         $s \leftarrow \tau^+ * s$      ▷ Expand trust region<br>
8:         **break**<br>
9:       **else**<br>
10:         $s \leftarrow \tau^- * s$      ▷ Shrink trust region<br>
11:       **if** $s <$ xtol **then**<br>
12:         **goto** 15<br>
13:     **if** converged according to tolerances xtol or ftol **then**<br>
14:       **break**<br>
15:   **if** constraints satisfied to tolerance ctol **then**<br>
16:     **break**<br>
17:   **else**<br>
18:     $\mu \leftarrow k * \mu$

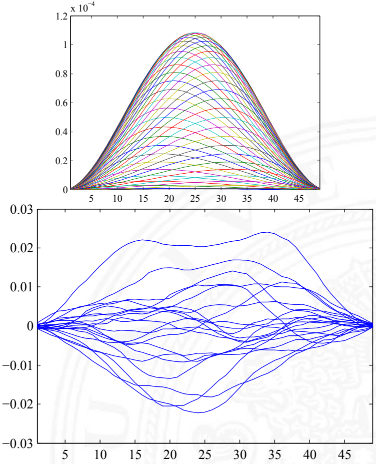
Source: [5]

# STOMP

- random sample based
- Algorithm:
  - initial trajectory
  - generate multiple noisy trajectories
  - evaluate the cost function
  - compute probability
  - update the through a weighted average
- non-differentiable

# STOMP: Noise

- $\tilde{u}_k = u + \epsilon_k$
- $\epsilon_k = \mathcal{N}(0, R^{-1})$
- Covariance dependent on finite difference matrix



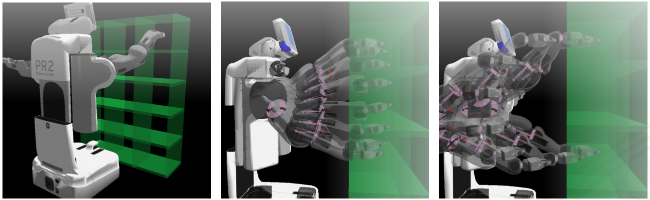

Source: [3]

# STOMP: Costs and Limits



- Obstacle Costs
  - like CHOMP
- Constraint Costs
  - end-effector position and orientation
- Torque Costs
  - requires dynamics model
- Joint Limits
  - clipping noisy trajectories

# STOMP: Comparison



| Scenario | STOMP Unconstrained | CHOMP Unconstrained | STOMP Constrained |
|---|---|---|---|
| Number of successful plans | 210 / 210 | 149 / 210 | 196 / 210 |
| Avg. planning time to success (sec) | 0.88 ± 0.40 | 0.71 ± 0.25 | 1.86 ± 1.25 |
| Avg. iterations to success | 52.1 ± 26.6 | 167.1 ± 113.8 | 110.1 ± 78.0 |

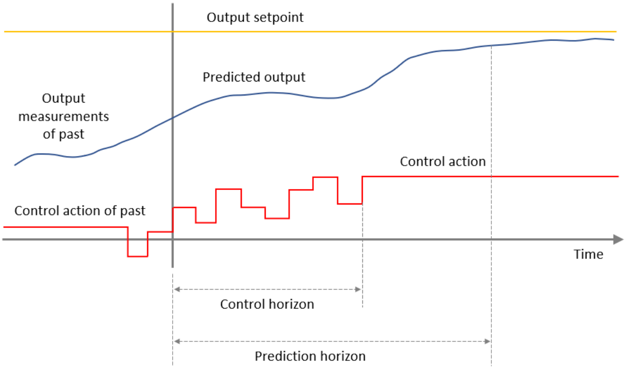
Source: [3]

# MPC

- Model Predictive Control
- predict system into future
- optimize actions $u$
- only apply first action $u_1$
- recompute often
- react to inaccuracies

# MPC: Horizon

- prediction for $[t, t + T_p]$
- control for $[t, t + T_c]$
- receding horizon




Source: [1]

# MPC: System Model

- Requires system Model
- must be linear
- $x_{t+1} = Ax_t + Bu_t$
- many systems are non-linear
  - approximately linear
  - solve non-linear model predictive control (NMPC)

# Comparison



- CHOMP
  - smooth
  - local minima
- TrajOpt
  - higher success ratio
  - local minima
- STOMP
  - no local minima
  - higher success ratio
  - arbitrary constrains
- MPC
  - real-time
  - requires system model
  - local minima

# Bibliography

[1] Run field oriented control of pmsm using model predictive control.
https://de.mathworks.com/help/examples/mcb/win64/xxmcb-mpc-pmsm-plot.png, 2025.
Last Accessed: 15.05.2025.

[2] forbs.
The-best-examples-of-human-and-robot-collaboration.
https://imageio.forbes.com/specials-images/imageserve/62f33bc82e274eb23360c8b8/The-Best-Examples-Of-Human-And-Robot-Collaboration/960x0.jpg?height=399&width=711&fit=bounds.
Last Accessed: 15.05.2025.

# Bibliography (cont.)

[3] Mrinal Kalakrishnan, Sachin Chitta, Evangelos Theodorou, Peter Pastor, and Stefan Schaal.
Stomp: Stochastic trajectory optimization for motion planning.
In *2011 IEEE International Conference on Robotics and Automation*, pages 4569–4574, 2011.

[4] researchleap.
Ai drive reasoning.
https://researchleap.com/wp-content/uploads/2021/12/AI_Drive_Reasoning-002.png, 2021.
Last Accessed: 15.05.2025.

[5] John Schulman, Jonathan Ho, Alex X Lee, Ibrahim Awwal, Henry Bradlow, and Pieter Abbeel.
Finding locally optimal, collision-free trajectories with sequential convex optimization.
In *Robotics: science and systems*, volume 9, pages 1–10. Berlin, Germany, 2013.

# Bibliography (cont.)

[6] Matt Zucker, Nathan Ratliff, Anca D. Dragan, Mihail Pivtoraiko, Matthew Klingensmith, Christopher M. Dellin, J. Andrew Bagnell, and Siddhartha S. Srinivasa.
Chomp: Covariant hamiltonian optimization for motion planning.
*The International Journal of Robotics Research*, 32(9-10):1164–1193, 2013.