Introduction to Robotics

Lecture 4

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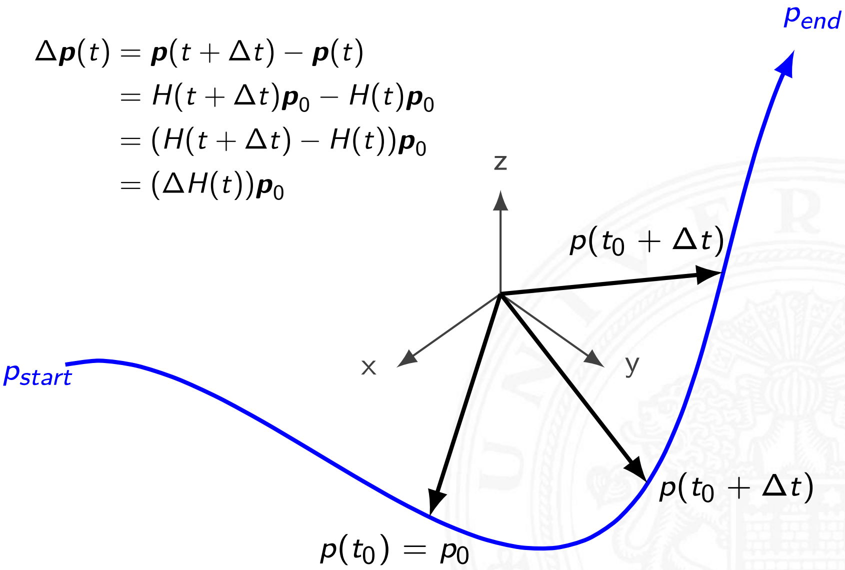
Outline

Outline (cont.)

Differential motion

$$\begin{aligned}\Delta \boldsymbol{p}(t) &= \boldsymbol{p}(t+\Delta t) - \boldsymbol{p}(t)\\ &= H(t+\Delta t)\boldsymbol{p}_0 - H(t)\boldsymbol{p}_0\\ &= (H(t+\Delta t) - H(t))\boldsymbol{p}_0\\ &= (\Delta H(t))\boldsymbol{p}_0\end{aligned}$$



Differential motion (cont.)

H is a 4×4 homogeneous transformation from world-frame to object-frame and $\boldsymbol{p}_0$ is given with reference to the world-frame.

Hence it is:

$$\dot{\boldsymbol{p}}(t) = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{p}(t)}{\Delta t} \tag{30}$$

$$= \frac{dH(t)}{dt} \mathbf{p_0} \tag{31}$$

$$= \left(\frac{dH(t)}{dt} H^{-1}(t) \right) H(t) \mathbf{p_0} \tag{32}$$

$$= \left(\frac{dH(t)}{dt} H^{-1}(t) \right) \mathbf{p}(t) \tag{33}$$

Derivative of a homogeneous transformation

Consider the homogeneous transformation H

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where each element is a function of a variable t:

$$dH = \begin{bmatrix} \frac{\partial h_{11}}{\partial t} & \frac{\partial h_{12}}{\partial t} & \frac{\partial h_{13}}{\partial t} & \frac{\partial h_{14}}{\partial t} \\ \frac{\partial h_{21}}{\partial t} & \frac{\partial h_{22}}{\partial t} & \frac{\partial h_{23}}{\partial t} & \frac{\partial h_{24}}{\partial t} \\ \frac{\partial h_{31}}{\partial t} & \frac{\partial h_{32}}{\partial t} & \frac{\partial h_{33}}{\partial t} & \frac{\partial h_{34}}{\partial t} \\ 0 & 0 & 0 & 1 \end{bmatrix} dt$$

Differential translation and rotation – World-frame

Case 1 The differential translation and rotation are executed with reference to a fixed coordinate frame.

$$H + dH = Trans_{dx,dy,dz} Rot_{k,d\theta}\ H \tag{34}$$

$Trans_{dx,dy,dz}$: is a differential translation dz, dy, dz with reference to the fixed coordinate frame.

$Rot_{k,d\theta}$: is a differential rotation $d\theta$ around an arbitrary vector **k** with reference to the fixed coordinate frame.

dH is calculated as follows:

$$dH = (Trans_{dx,dy,dz} Rot_{k,d\theta} - I)\ H \tag{35}$$

Differential translation and rotation – Object-frame

Case 2 The differential translation and rotation are executed with reference to a current object coordinate frame:

$$H + dH = H \; Trans_{dx,dy,dz} Rot_{k,d\theta} \tag{36}$$

$Trans_{dx,dy,dz}$: is a differential translation dz, dy, dz with reference to the current object coordinate frame.

$Rot_{k,d\theta}$: is a differential rotation $d\theta$ around an arbitrary vector **k** with reference to the current object coordinate frame.

dH is calculated as follows:

$$dH = H \; (Trans_{dx,dy,dz} Rot_{k,d\theta} - I) \tag{37}$$

Differential homogeneous transformation

Definition

$$\boldsymbol{\Delta} = Trans_{dx,dy,dz} Rot_{k,d\theta} - I$$

Thus (35) can be written as

$$dH = \boldsymbol{\Delta} \cdot H$$

and (37) can be written as:

$$dH = H \cdot \boldsymbol{\Delta}$$

Differential homogeneous transformation (cont.)

The translation by **d** is defined as:

$$Trans_{\boldsymbol{d}} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\boldsymbol{d}$ is a differential vector that represents the differential change

$$d_x \overrightarrow{i} + d_y \overrightarrow{j} + d_z \overrightarrow{k}$$

($\overrightarrow{i}$, $\overrightarrow{j}$, $\overrightarrow{k}$ are three unit vectors coinciding with x, y, z).

Differential homogeneous transformation (cont.)

The transformation of the rotation with θ around an arbitrary vector $\boldsymbol{k} = k_x \overrightarrow{i} + k_y \overrightarrow{j} + k_z \overrightarrow{k}$ is defined as:

$$Rot_{\boldsymbol{k},\theta} = \begin{bmatrix} k_x k_x V\theta + C\theta & k_y k_x V\theta - k_z S\theta & k_z k_x V\theta + k_y S\theta & 0 \\ k_x k_y V\theta + k_z S\theta & k_y k_y V\theta + C\theta & k_z k_y V\theta - k_x S\theta & 0 \\ k_x k_z V\theta - k_y S\theta & k_y k_z V\theta + k_x S\theta & k_z k_z V\theta + C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{38}$$

where $C\theta = \cos\theta$, $S\theta = \sin\theta$
and $V\theta = \text{versine}\,\theta = 2\sin^2(\frac{\theta}{2}) = 1 - \cos\theta$.

see R. Paul, *Robot Manipulators: Mathematics, Programming, and Control: the Computer Control of Robot Manipulators*. Artificial Intelligence Series, MIT Press, 1981, section 1.12 "General Rotation Transformation"

Differential homogeneous transformation (cont.)

With:

$$\lim_{\theta \to 0} \sin\theta \to d\theta$$

$$\lim_{\theta \to 0} \cos\theta \to 1$$

$$\lim_{\theta \to 0} vers\theta \to 0$$

(38) can be written as:

$$Rot_{\mathbf{k},\theta} = \begin{bmatrix} 1 & -k_z d\theta & k_y d\theta & 0 \\ k_z d\theta & 1 & -k_x d\theta & 0 \\ -k_y d\theta & k_x d\theta & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (39)$$

Differential homogeneous transformation (cont.)

$$\boldsymbol{\Delta} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -k_z d\theta & k_y d\theta & 0 \\ k_z d\theta & 1 & -k_x d\theta & 0 \\ -k_y d\theta & k_x d\theta & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{40}$$

$$= \begin{bmatrix} 0 & -k_z d\theta & k_y d\theta & d_x \\ k_z d\theta & 0 & -k_x d\theta & d_y \\ -k_y d\theta & k_x d\theta & 0 & d_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{41}$$

Differential rotation around the x,y,z axes

Rotation matrices for rotations around x, y and z axis

$$R_{x,\psi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\psi & -S\psi & 0 \\ 0 & S\psi & C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{42}$$

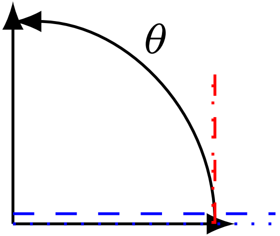
$$R_{y,\theta} = \begin{bmatrix} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{43}$$

$$R_{z,\phi} = \begin{bmatrix} C\phi & -S\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{44}$$

Differential rotation around the x,y,z axes (cont.)

Considering the differential change:

$sin\theta \rightarrow \delta\theta$ and
$cos\theta \rightarrow 1.$



$$R_{x,\delta_x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\delta_x & 0 \\ 0 & \delta_x & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (45)$$

$$R_{y,\delta_y} = \begin{bmatrix} 1 & 0 & \delta_y & 0 \\ 0 & 1 & 0 & 0 \\ -\delta_y & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (46)$$

$$R_{z,\phi} = \begin{bmatrix} 1 & -\delta_z & 0 & 0 \\ \delta_z & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (47)$$

Differential rotation around the x,y,z axes (cont.)

Omitting terms of the 2nd order, one gets:

$$R_{z,\delta_z} R_{y,\delta_y} R_{x,\delta_x} = \begin{bmatrix} 1 & -\delta_z & \delta_y & 0 \\ \delta_z & 1 & -\delta_x & 0 \\ -\delta_y & \delta_x & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{48}$$

Through comparison of (39) with (48) one determines:

$$k_x d\theta = \delta_x \tag{49}$$

$$k_y d\theta = \delta_y \tag{50}$$

$$k_z d\theta = \delta_z \tag{51}$$

Differential rotation around the x,y,z axes (cont.)

Equation (41) can be rewritten as:

$$\boldsymbol{\Delta} = \begin{bmatrix} 0 & -\delta_z & \delta_y & d_x \\ \delta_z & 0 & -\delta_x & d_y \\ -\delta_y & \delta_x & 0 & d_z \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Definition of differential transformation

$\boldsymbol{\Delta}$ is therefore fully defined by the vectors $\boldsymbol{d}$ and $\boldsymbol{\delta}$.

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