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MIN Faculty
Department of Informatics

Introduction to Robotics

Lecture 2

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Technical Aspects of Multimodal Systems

April 12, 2018

Outline

Outline (cont.)

Forward kinematics

- Movement depiction of mechanical systems
- Here, only position is addressed
- Translate a series of joint parameter to cartesian position
- Depiction of the mechanical system as fixed body chain
  - Serial robots
- Types of joints
  - rotational joints
  - prismatic joints

Mitsubishi PA10-6C

Kinematics

- Transformation regulation, which describes the relation between joint coordinates of a robot **q** and the environment coordinates of the endeffector **x**
- Solely determined by the geometry of the robot
  - Base frame
  - Relation of frames to one another
    $\Longrightarrow$ Formation of a recursive chain
  - Joint coordinates:

$$q_i = \begin{cases} \theta_i & : \text{rotational joint} \\ d_i & : \text{translation joint} \end{cases}$$

Purpose

Absolute determination of the position of the endeffector (TCP) in the cartesian coordinate system

Kinematic equations

- Manipulator is considered as set of links connected by joints
- In each link, a coordinate frame is defined
- A homogeneous matrix ${}^{i-1}A_i$ depicts the relative translation and rotation between two consecutive joints
    - Joint transition

Kinematic equations (cont.)

For a manipulator consisting of six joints:

- ${}^{0}A_1$: depicts position and orientation of the first link
- ${}^{1}A_2$: position/orientation of the 2nd link with respect to link 1

$$\vdots$$

- ${}^{5}A_6$: depicts position and orientation of the 6th link in regard to link 5

The resulting product is defined as:

$$T_6 = {}^{0}A_1 \, {}^{1}A_2 \, {}^{2}A_3 \, {}^{3}A_4 \, {}^{4}A_5 \, {}^{5}A_6$$

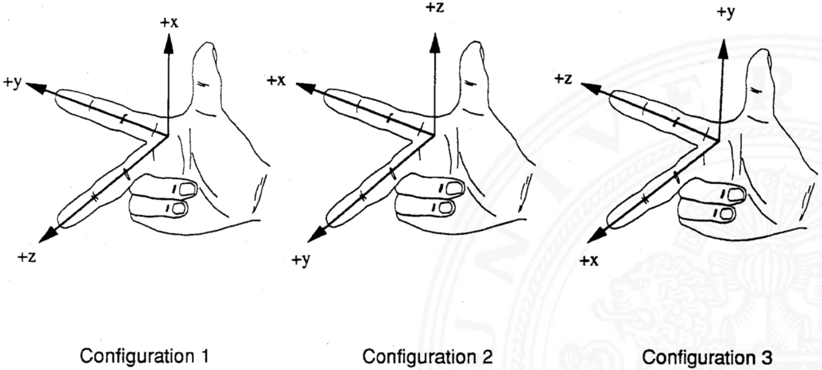
Kinematic description

- Calculation of $T_6 = \prod_{i=1}^{n} A_i$ A_i short for ${}^{i-1}A_i$
  - T_6 defines, how n joint transitions describe 6 cartesian DOF
- Definition of one coordinate system (CS) per segment i
  - generally arbitrary definition
- Determination of one transformation A_i per segment $i = 1..n$
  - generally 6 parameters (3 rotational + 3 translational) required
  - different sets of parameters and transformation orders possible

Solution

Denavit-Hartenberg (DH) convention

Right-Handed Coordinate System



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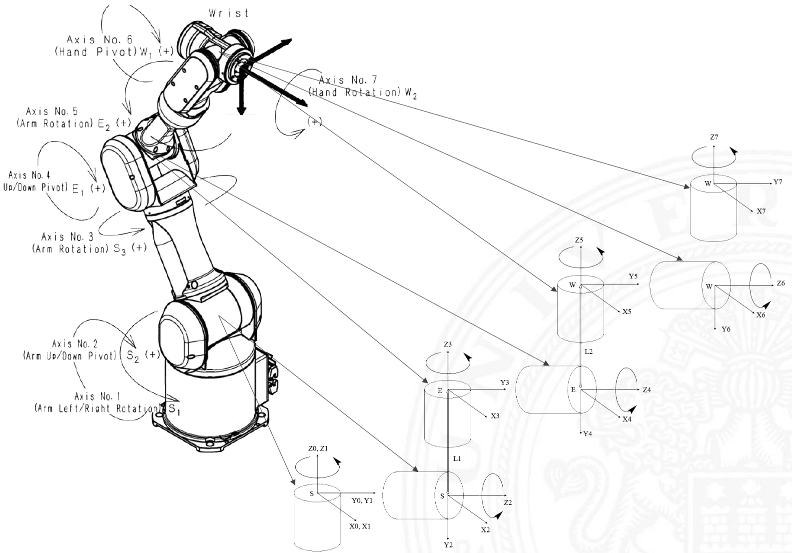


Tool Center Point (TCP) description

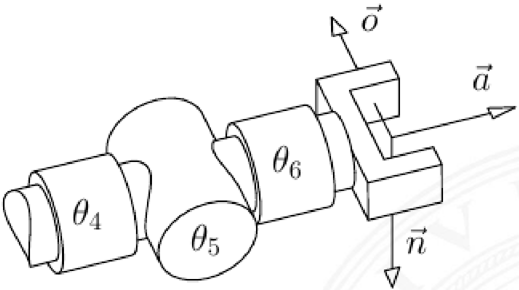
Using a vector $\vec{p}$, the TCP position is depicted.

Three unit vectors:

- $\vec{a}$: (approach vector),
- $\vec{o}$: (orientation vector),
- $\vec{n}$: (normal vector)

specify the orientation of the TCP.

Tool Center Point (TCP) description (cont.)



Thus, the transformation T consists of the following elements:

$$T = \begin{bmatrix} \vec{n} & \vec{o} & \vec{a} & \vec{p} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Denavit-Hartenberg convention

- first published by Denavit and Hartenberg in 1955
- established principle
- determination of a transformation matrix A_i using **four** parameter
  - joint length, joint twist, joint offset and joint angle (a_i, α_i, d_i, θ_i)
- complex transformation matrix A_i results from four atomic transformations

Transformation order

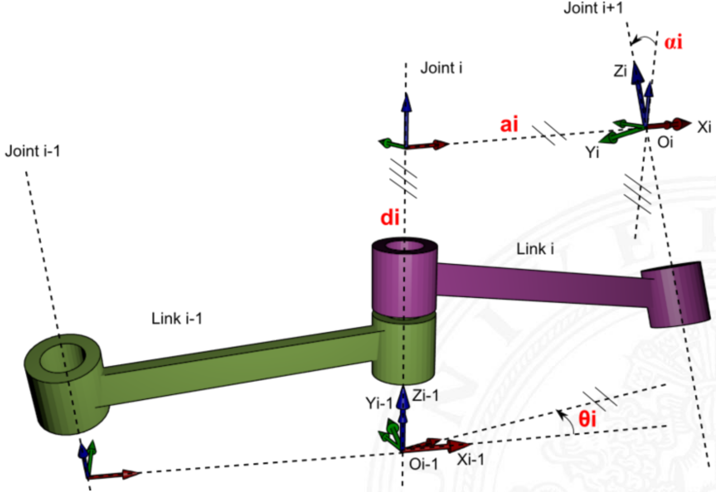
Classic:

$$A_i = R_{z_{i-1}}(\theta_i) \cdot T_{z_{i-1}}(d_i) \cdot T_{x_i}(a_i) \cdot R_{x_i}(\alpha_i) \rightarrow CS_i$$

Modified:

$$A_i = R_{x_{i-1}}(\alpha_{i-1}) \cdot T_{x_{i-1}}(a_{i-1}) \cdot R_{z_i}(\theta_i) \cdot T_{x_i}(d_i) \rightarrow CS_i$$

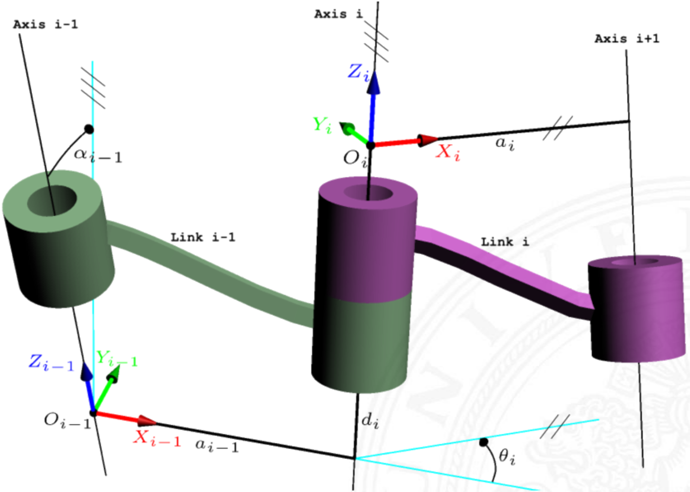
Classic Parameters



Transformation order

$$A_i = R_{z_{i-1}}(\theta_i) \cdot T_{z_{i-1}}(d_i) \cdot T_{x_i}(a_i) \cdot R_{x_i}(\alpha_i) \rightarrow CS_i$$

Modified Parameters



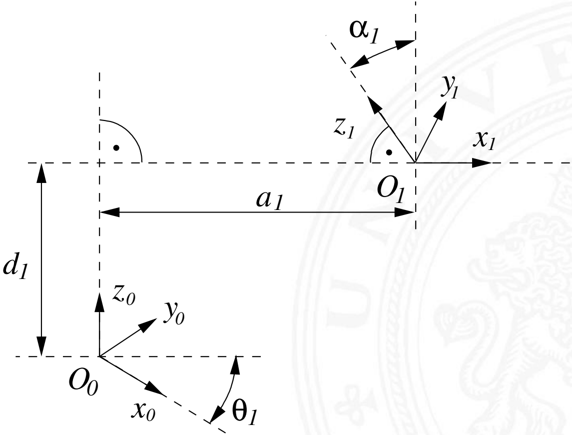
Transformation order

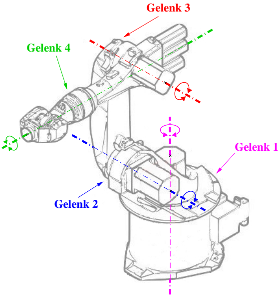
$$A_i = R_{x_{i-1}}(\alpha_{i-1}) \cdot T_{x_{i-1}}(a_{i-1}) \cdot R_{z_i}(\theta_i) \cdot T_{x_i}(d_i) \rightarrow CS_i$$

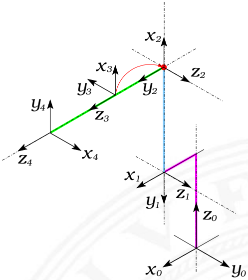
DH-Parameters and -Preconditions (classic)

Idea: Determination of the transformation matrix A_i using four joint parameters (a_i, α_i, d_i, θ_i) and two preconditions

- DH$_1$ x_i is perpendicular to z_{i-1}
- DH$_2$ x_i intersects z_{i-1}







- CS_0 is the stationary origin at the base of the manipulator
- axis z_{i-1} is set along the axis of motion of the i^{th} joint
- axis x_i is parallel to the common normal of z_{i-1} and z_i $(x_i \parallel (z_{i-1} \times z_i))$.
- axis y_i concludes a right-handed coordinate system

Frame transformation for two links (classic)

Creation of the relation between frame i and frame $(i-1)$ through the following rotations and translations:

- Rotate around z_{i-1} by angle θ_i
- Translate along z_{i-1} by d_i
- Translate along x_i by a_i
- Rotate around x_i by angle α_i

Using the product of four homogeneous transformations, which transform the coordinate frame $i-1$ into the coordinate frame i, the matrix A_i can be calculated as follows:

$$A_i = R_{z_{i-1}}(\theta_i) \cdot T_{z_{i-1}}(d_i) \cdot T_{x_i}(a_i) \cdot R_{x_i}(\alpha_i) \rightarrow CS_i$$

Frame transformation for two links (classic) (cont.)

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dots & 0 \\ \dots & 0 \\ \dots & d_i \\ \dots & 1 \end{bmatrix} \begin{bmatrix} \dots & a_i \\ \dots & 0 \\ \dots & 0 \\ \dots & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- using DH$_1$ $x_1 \cdot z_0 = 0$

$$0 = {}^0x_1 \cdot {}^0z_0 \tag{2}$$

$$0 = \left({}^0A_1 x_1\right)^T \cdot z_0 \tag{3}$$

$$= \left(\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)^T \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

$$= \begin{bmatrix} r_{11} & r_{21} & r_{31} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

$$= r_{31} \tag{6}$$

$$\Longrightarrow \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ 0 & r_{32} & r_{33} \end{bmatrix}$$

Background of DH-convention (cont.)

- with ${}^{i-1}R_i$ being orthogonal and orthonormal

$$r_{11}^2 + r_{21}^2 = 1 \tag{7}$$

$$r_{32}^2 + r_{33}^2 = 1 \tag{8}$$

$(r_{11}, r_{21}) = (\cos\theta, \sin\theta)$ and
$(r_{32}, r_{33}) = (\sin\alpha, \cos\alpha)$
fulfill the constraint;

$$\Rightarrow \begin{bmatrix} \cos\theta & r_{12} & r_{13} \\ \sin\theta & r_{22} & r_{23} \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

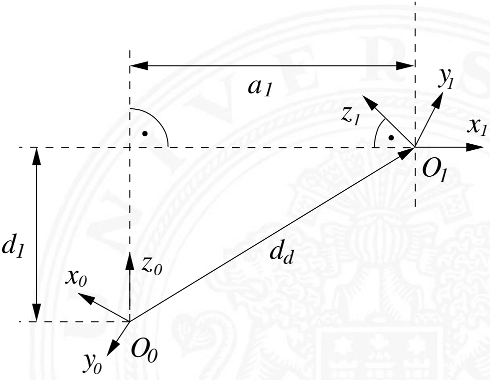
- r_{12}, r_{13}, r_{22} and r_{23} can complete the rotational matrix

$$\Rightarrow \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix}$$

Background of DH-convention (cont.)

- with DH$_2$ and DH$_1$:
  the positional vector d_d from O_0 to O_1 may be represented as a linear combination of vectors z_0 and x_1

$$
\begin{aligned}
{}^0d_d &= d\, z_0 + a\ {}^0A_1 x_1 \\
&= d \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + a\ {}^0R_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\
&= d \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + a \begin{bmatrix} cos\theta \\ sin\theta \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} a\, cos\theta \\ a\, sin\theta \\ d \end{bmatrix}
\end{aligned}
$$



Background of DH-convention (cont.)

- homogeneous transformation A_i fulfills DH$_2$ and DH$_1$

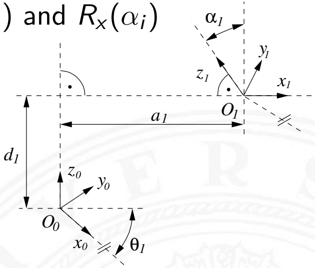
$$A_i = R_z(\theta_i) \cdot T_z(d_i) \cdot T_x(a_i) \cdot R_x(\alpha_i)$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Partial order of transformation

Calculation of homogeneous transformation matrix A_1 from the partial transformations $R_z(\theta_i)$, $T_z(d_i)$, $T_x(a_i)$ and $R_x(\alpha_i)$



- transition CS_0 to CS_1 using local axes
- invariances
  - T_x invariant to R_x ($T_x\, R_x = R_x\, T_x$)
  - T_z invariant to R_z ($T_z\, R_z = R_z\, T_z$)
- order of transformations
  - rotation around z_1 after rotation around x_0 violates DH$_2$
  - thus, possible rotation orders which do not violate DH$_2$ and DH$_1$:

$$A_i = R_{x_1'''}(\alpha_1) \cdot T_{x_1''}(a_1) \cdot T_{z_0'}(d_1) \cdot R_{z_0}(\theta_1) \qquad (9)$$

$$= R_{z_0}(\theta_i) \cdot T_{z_0}(d_i) \cdot T_{x_1}(a_i) \cdot R_{x_1}(\alpha_i) \qquad (10)$$

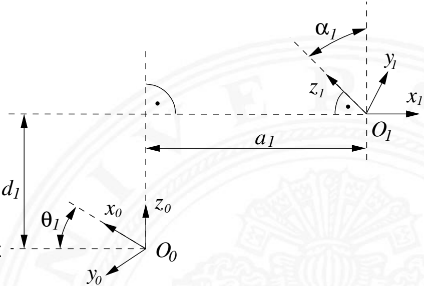
(9) is a possible valid transformation order
(10) is the standard transformation order

Definition of joint coordinate systems: Exceptions

Beware

The Denavit-Hartenberg convention is not unambiguous!

- z_{i-1} is parallel to z_i
  - arbitrary shortest normal
  - usually $d_i = 0$ is chosen
- z_{i-1} intersects z_i
  - usually $a_i = 0$ such that CS lies in the intersection point

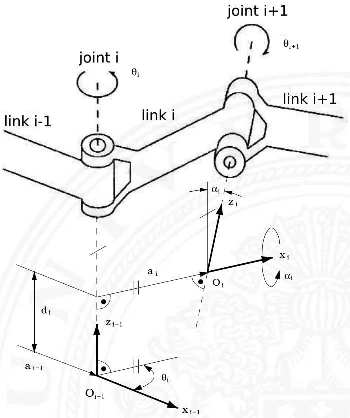


- orientation of CS_n ambigous, as no joint $n+1$ exists
  - x_n must be a normal to z_{n-1}
  - usually z_n chosen to point in the direction of the approach vector $\vec{a}$ of the tcp

Parameters for description of two arbitrary links

Two parameters for the description of the link structure i

- a_i : shortest distance between the z_{i-1}-axis and the z_i-axis
- α_i : rotation angle around the x_i-axis, which aligns the z_{i-1}-axis to the z_i-axis

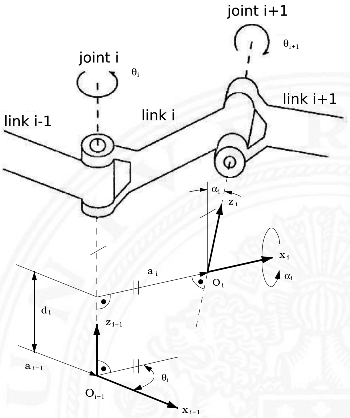


a_i and α_i are constant values due to construction

Parameters for describing two arbitrary links (cont.)

Two for relative distance and angle of adjacent links

- d_i: distance origin O_{i-1} of the $(i\text{-}1)^{st}$ CS to intersection of z_{i-1}-axis with x_i-axis
- θ_i: joint angle around z_{i-1}-axis to align x_{i-1}-parallel to x_i-axis into x_{i-1}, y_{i-1}-plane



θ_i and d_i are variable

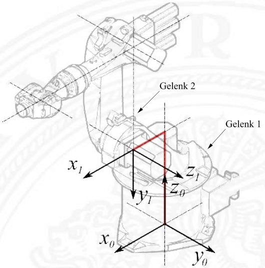
- rotational: θ_i variable, d_i fixed
- translational: d_i variable, θ_i fixed

Example DH-Parameter of a single joint

Determination of DH-Parameter (θ, d, a, α) for calculation of joint transformation: $A_1 = R_z(\theta_1) T_z(d_1) T_x(a_1) R_x(\alpha_1)$

joint angle rotate by θ_1 around z_0, such that x_0 is parallel to x_1

$$R_z(\theta_1) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

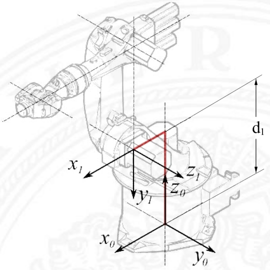


for the shown joint configuration $\theta_1 = 0°$

Example DH-Parameter of a single joint (cont.)

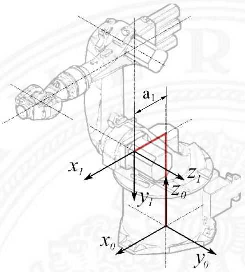
joint offset translate by d_1 along z_0 until the intersection of z_0 and x_1

$$T_z(d_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Example DH-Parameter of a single joint (cont.)

joint length translate by a_1 along x_1 such that the origins of both CS are congruent

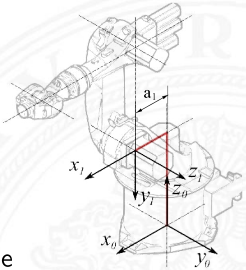


$$T_x(a_1) = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example DH-Parameter of a single joint (cont.)

joint twist rotate z_0 by α_1 around x_1, such that z_0 lines up with z_1

$$R_{x(\alpha_1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_1) & -\sin(\alpha_1) & 0 \\ 0 & \sin(\alpha_1) & \cos(\alpha_1) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



for the shown joint configuration, $\alpha_1 = -90^\circ$ due to construction

Example DH-Parameter of a single joint (cont.)

- total transformation of CS_0 to CS_1 (general case)

$$
\begin{aligned}
{}^0A_1 &= R_z(\theta_1) \cdot T_z(d_1) \cdot T_x(a_1) \cdot R_x(\alpha_1) \\
&= \begin{bmatrix} cos\theta_1 & -sin\theta_1 cos\alpha_1 & sin\theta_1 sin\alpha_1 & a_1 cos\theta_1 \\ sin\theta_1 & cos\theta_1 cos\alpha_1 & -cos\theta_1 sin\alpha_1 & a_1 sin\theta_1 \\ 0 & sin\alpha_1 & cos\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

- rotary case: variable θ_1 and fixed d_1, a_1 *und* $(\alpha_1 = -90°)$

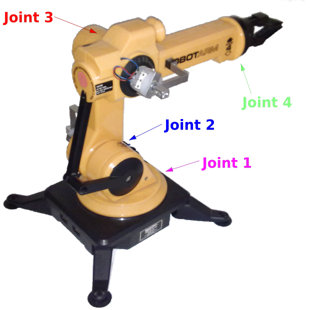
$$
\begin{aligned}
{}^0A_1 &= R_z(\theta_1) \cdot T_z(d_1) \cdot T_x(a_1) \cdot R_x(-90°) \\
&= \begin{bmatrix} cos\theta_1 & 0 & -sin\theta_1 & a_1 cos\theta_1 \\ sin\theta_1 & 0 & cos\theta_1 & a_1 sin\theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

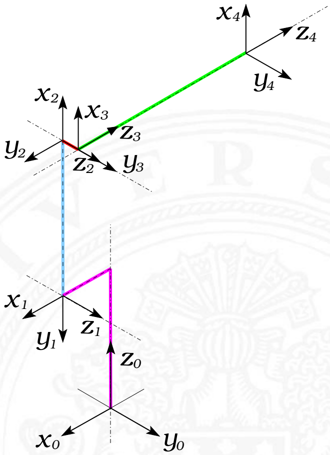
Procedure for predefined structure

- Fixed origin: CS_0 is the fixed frame at the base of the manipulator
- Determination of axes and consecutive numbering from 1 to n
- Positioning O_i on rotation- or shear-axis i, z_i points aways from z_{i-1}
- Determination of normal between the axes; setting x_i (in direction to the normal)
- Determination of y_i (right-hand system)
- Read off Denavit-Hartenberg parameter
- Calculation of overall transformation

Example DH-Parameter for Quickshot

- Definition of CS corresponding to DH convention
- Determination of DH-Parameter





Example Transformation matrix T_6

$$T_6 = A_1 \cdot A_2 \cdot A_3 \cdot A_4$$

$$= \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 20\cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & 20\sin\theta_1 \\ 0 & -1 & 0 & 100 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 160\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 160\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot$$

$$\begin{bmatrix} \cos\theta_3 & 0 & \sin\theta_3 & 0 \\ \sin\theta_3 & 0 & -\cos\theta_3 & 0 \\ 0 & 1 & 0 & 28 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 250 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1\cos\theta_4(\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3) - \sin\theta_1\sin\theta_4 & \dots & \dots & \dots \\ \sin\theta_1\cos\theta_4(\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3) + cos\theta_1\sin\theta_4 & \dots & \dots & \dots \\ -\cos\theta_4(\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3) & \dots & \dots & \dots \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example featuring PUMA 560

In order to transfer the manipulator-endpoint into the base coordinate system, T_6 is calculated as follows:

$$T_6 = A_1 A_2 A_3 A_4 A_5 A_6$$

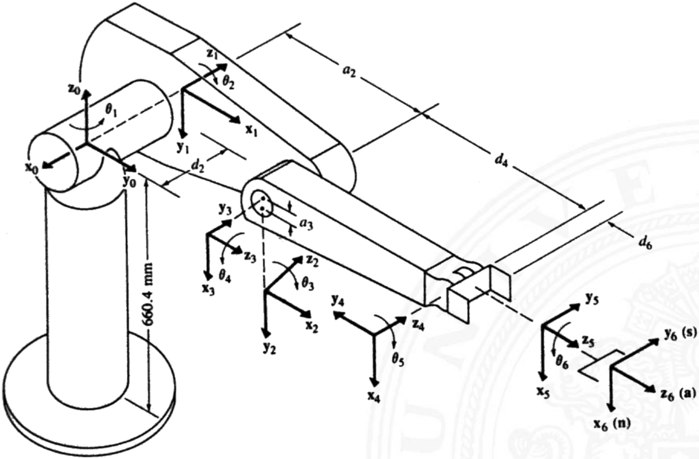
Z: Transformation manipulator base $\rightarrow$ reference coordinate system
E: Manipulator endpoint $\rightarrow$ TCP ("tool center point")
X: The position and orientation of the TCP in relation of the reference coordinate system

$$X = Z T_6 E$$

The following applies as well:

$$T_6 = Z^{-1} X E^{-1}$$

Example featuring PUMA 560 (cont.)



Link Transformations

$$T_6^0 =^0 T_1^1 T_2^2 T_3^3 T_4^4 T_5^5 T_6$$

$$^0T_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S\theta_2 & -C\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link Transformations (cont.)

$$
{}^{2}T_{3} = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_2 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}T_{4} = \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -S\theta_4 & -C\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Link Transformations (cont.)

$$
{}^4T_5 = \begin{bmatrix} C\theta_5 & -S\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S\theta_5 & -C\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^4T_5 = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S\theta_6 & -C\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The solution using the example of PUMA 560

Sum-of-Angle formula

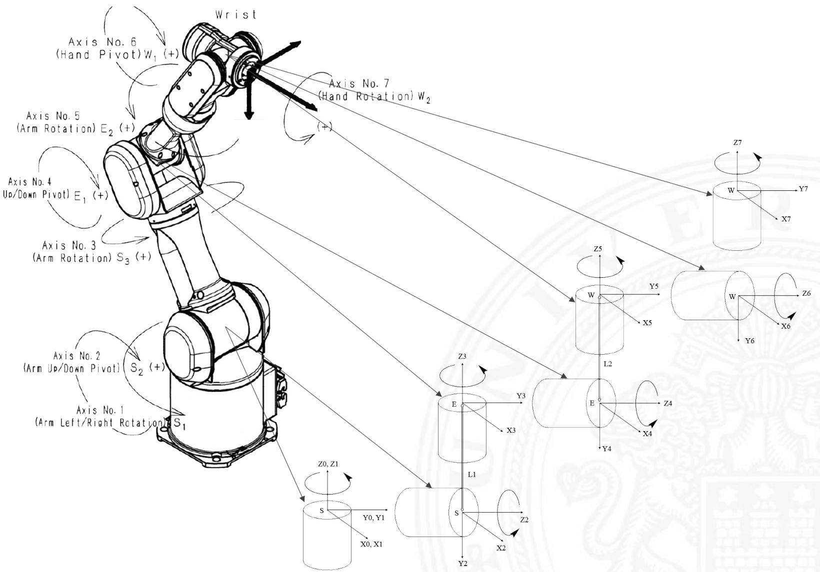
$C_{23} = C_2 C_3 - S_2 S_3,$
$S_{23} = C_2 S_3 + S_2 C_3$

$$T_6^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The solution using the example of PUMA 560 (cont.)

$$n_x = C_1[C_{23}(C_4C_5C_6 - S_4S_5) - S_{23}S_5C_5] - S_1(S_4C_5C_6 + C_4S_6)$$
$$n_y = S_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] + C_1(S_4C_5C_6 + C_4S_6)$$
$$n_z = -S_{23}[C_4C_5C_6 - S_4S_6] - C_{23}S_5C_6$$
$$o_x, o_y, o_z = \ldots$$
$$a_x, a_y, a_z = \ldots$$
$$p_x = C_1[a_2C_2 + a_3C_{23} - d_4S_{23}] - d_3S_1$$
$$p_y = S_1[a_2C_2 + a_3C_{23} - d_4S_{23}] + d_3C_1$$
$$p_z = -a_3S_{23} - a_2S_2 - d_4C_{23}$$

Mitsubishi PA10-7C



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