Surface Reconstruction with Alpha Shapes

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Outline

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Motivation
How to reconstruct a surface from a given set of points?

**INPUT**
range or contour data
(e.g. from laser range finder)

**OUTPUT**
(most optimal) approximation of the real surface

Point set [4]

Alpha Shape [4]
Motivation
The ice cream analogy

- ice cream with solid chocolate chips
- spherical ice spoon
- curve out all parts of the ice cream without touching the chocolate chips
- straighten all curvatures

Alpha Shape in 2-dimensional space [4]
Background:
How about the theory?

2D/3D
Explanation will be for 2D, extending to 3D is trivial
Background

k-simplex

Definition

k-simplex: Any subset $T \subseteq S$ of size $|T| = k + 1$, with $0 \leq k \leq 3(d)$ defines a $k$-simplex $\triangle_T$ that is the convex hull of $T$. [8]

http://kurlin.org/blog/complexes-are-discretizations-of-shapes/
Background

Simplicial complex

Definition

Simplicial complex:
A collection $C$ of simplices forms a simplicial complex if it satisfies the following conditions:

1. for a simplex $\Delta_T$ of $C$, the boundary simplices of $\Delta_T$ are in $C$
2. for two simplices of $C$, their intersection is either $\emptyset$ or a simplex in $C$

[5]
Background

**Delaunay triangulation**

**Problem**

- Given: point set $S$
- Underlying space: convex hull of $S$
- Goal: Divide $\text{conv}(S)$ into triangles with points of $S$ as vertices.
Background
Delaunay triangulation (cont.)

Algorithm

For each subset $T \subseteq S$, with $|T| = 3$
1. Test whether the circumcircle of $T$ is empty
2. If yes, the points of $T$ make up a triangle
3. otherwise discard $T$
Background
Delaunay triangulation (cont.)

Algorithm

For each subset $T \subseteq S$, with $|T| = 3$

1. Test whether the circumcircle of $T$ is empty
2. If yes, the points of $T$ make up a triangle
3. Otherwise discard $T$

Emptiness test is successful
Background

Delaunay triangulation (cont.)
The alpha complex $C_\alpha$ is a subcomplex of the Delaunay triangulation ($DT$)

Each $k$-simplex $\Delta_T \in DT(S)$ is in the alpha complex $C_\alpha$ if

(i) the circumcircle of $T$ with radius $r < \alpha$ is empty or

(ii) it is a boundary simplex of a simplex of (i)

The polytope $S_\alpha$ then is the underlying space (i.e. union of all $k$-simplices $\Delta_T$) of the alpha complex $C_\alpha$:

$$|C_\alpha| = S_\alpha$$
Alpha Shapes
Family

Family of $\alpha$-shapes $S_\alpha$ ($0 \leq \alpha \leq \infty$)

$\alpha = \{0, 0.19, 0.25, 0.75, \infty\}$ [10]

$S_0 = S$

$S_\infty = \text{conv}(S)$
Application in Robotics

Scene recovery and analysis

3D Scene Recovery and Spatial Scene Analysis for Unorganized Point Clouds [9]

- extracting spatial entities from point clouds
- region growing as segmentation method
- surface reconstructing of each region by alpha shapes
- properties of alpha shapes are used to infer semantics
Application in Robotics

Scene recovery and analysis

- Extracting spatial entities from point clouds
- Region growing as segmentation method
- Surface reconstructing of each region by alpha shapes
- Properties of alpha shapes are used to infer semantics
Problems & Limitations - Accuracy

- Choosing the ”best” $\alpha$ value is not trivial $\rightarrow$ some (heuristical) methods
- Not for all object’s surfaces there is a good $\alpha$ value due to non-uniformly sampled data
  - Interstices might be covered
  - Neighboring objects might be connected
  - Joints or sharp turns might not be sharp anymore

[10] E. Fließwasser
Problems & Limitations
Accuracy

Improvement: locally adjusting $\alpha$ test

- density scaling [10]
- anisotropic scaling [10]
- weighted alpha shapes [7]

Left: density scaling, right: added anisotropic scaling
Problems & Limitations

Time complexity

- Depends mostly on computation of Delaunay triangulation
- For DT in worst-case $O(n^2)$, with $n$ as number of points
- Edelsbrunner and Shar [6] developed a method for regular triangulations that performs with $O(n \log n)$. Mostly gives a complexity closer to linear. [10]
### Comparison

<table>
<thead>
<tr>
<th>Method</th>
<th>Time complexity</th>
<th>Robustness</th>
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<tbody>
<tr>
<td>Cocone Algorithm[1]</td>
<td>quadratic (based on Voronoi Filtering)</td>
<td>Noise: no; Undersampling: no</td>
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- There are (heuristical) methods that improve robustness for each algorithm.
- Especially for undersampling and non-uniform sampled data by local adaption.


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<th>Author(s) and Title</th>
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Surface reconstruction with anisotropic density-scaled alpha shapes.  