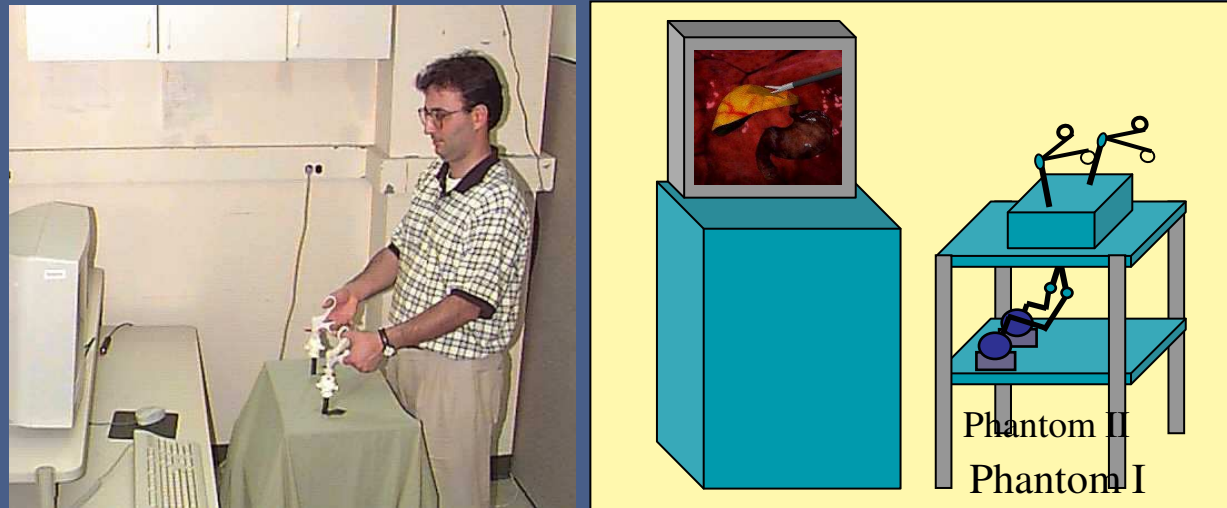


# Simulation of Surgical Procedures in Virtual Environments



**Cagatay Basdogan, Ph.D.**

College of Engineering,  
Koc University

Notes & Equations & Matlab Code are online:

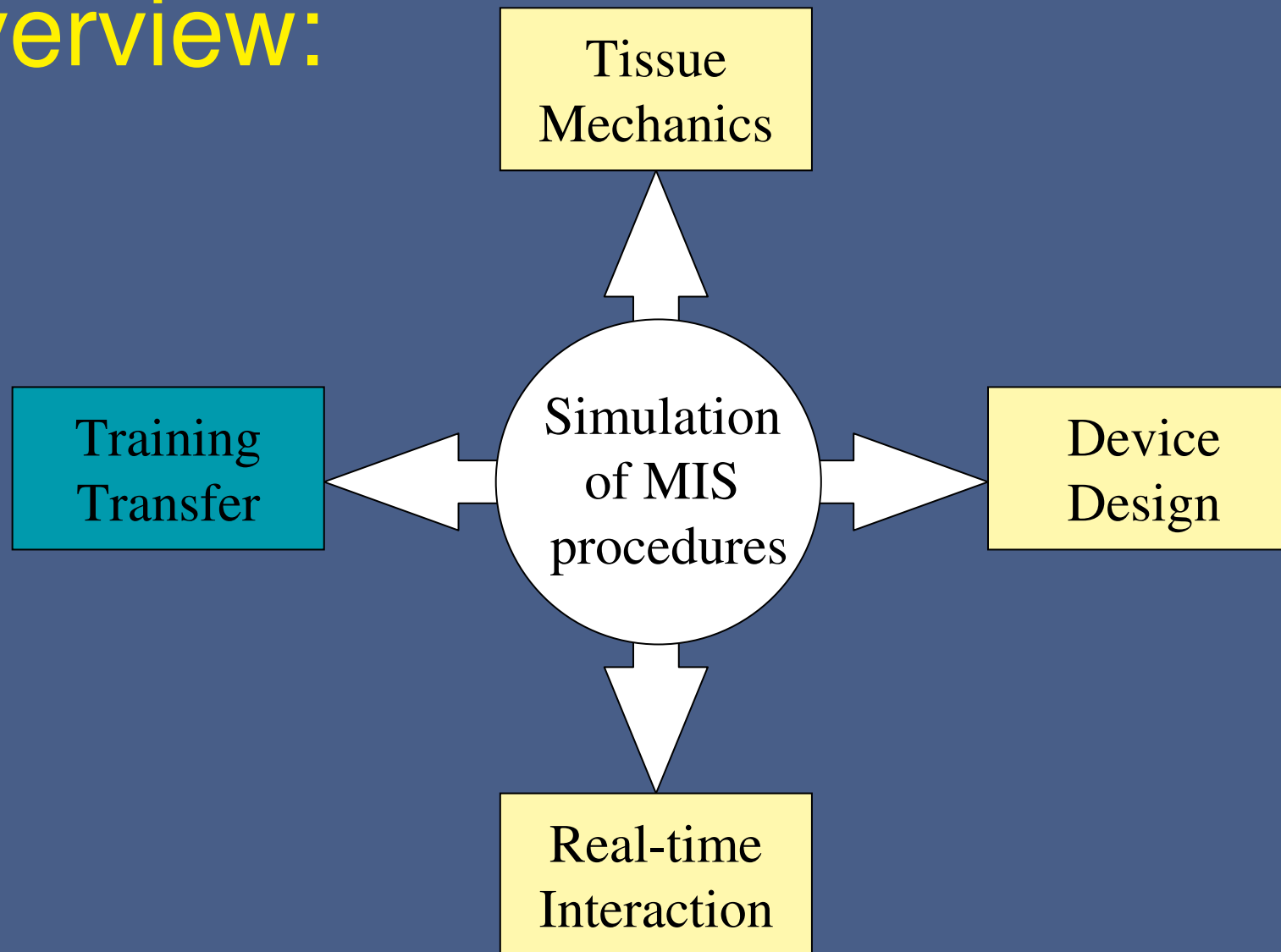
<http://network.ku.edu.tr/~basdogan>

# Significance:

## VR-based training

- to train and certify medical personnel
- to reduce the use of animals in training
- to test new surgical devices and procedures

# Overview:

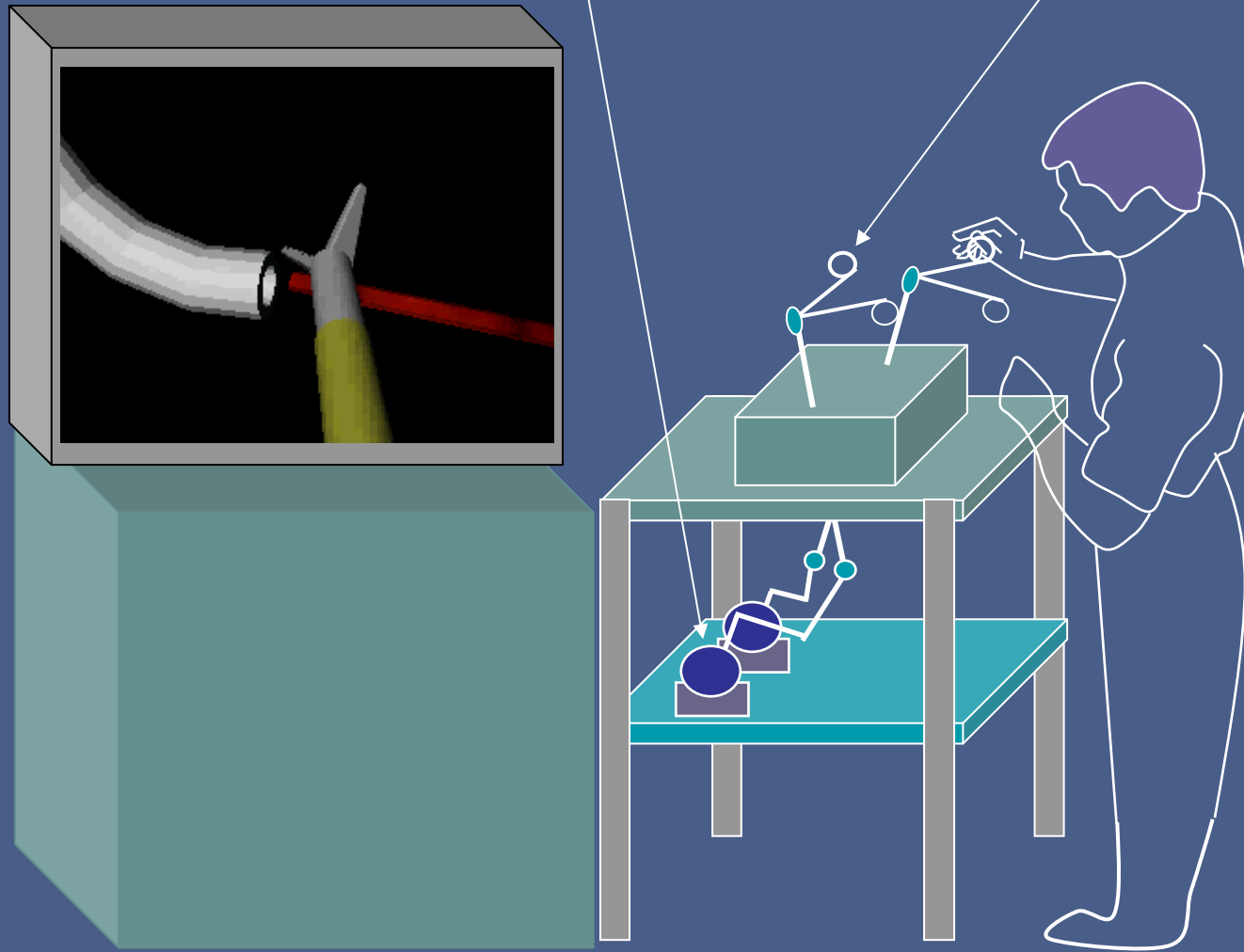


# System:

Visual  
Display

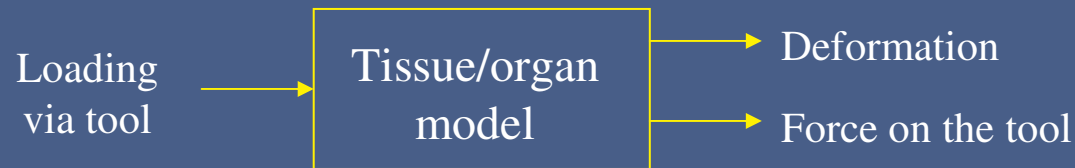
Haptic Displays

Laparoscopic  
Instruments



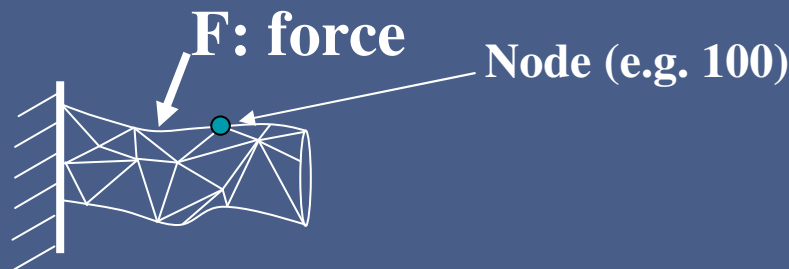
# Challenges:

## Physically-Based Modeling



**FEM ?**

$$F = M\ddot{U} + C\dot{U} + KU \quad (\text{dynamic analysis})$$



- membrane+bending elements
- 6-dof for each node
- Stiffness Matrix : K (600x600), diagonal M and C

→ **600 coupled equations !**

## Real-time Display

- Graphics (30 Hz)
- Haptics (1000 Hz)

→ **1 msec !**

## Tissue Characteristics

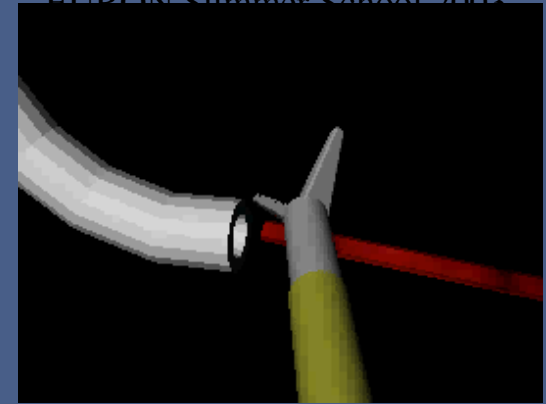
- nonlinear
- anisotropic
- hysteresis
- non-homogeneous

→ **complex !**

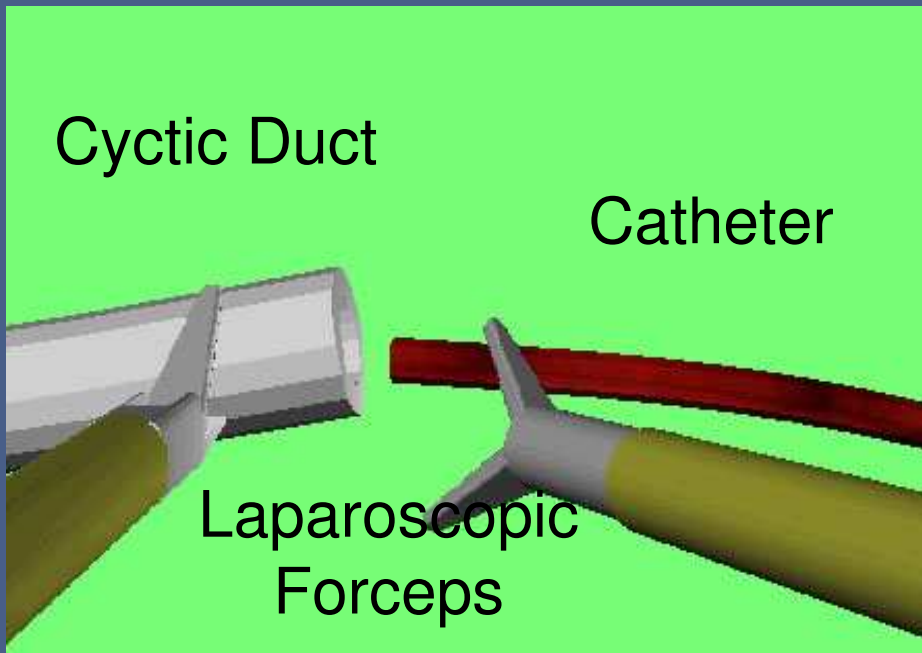
## **Topics:**

- A) Collision detection and computational models of surgical instruments**
- B) Physically-based modeling for simulating soft tissue behavior**
- C) Haptic rendering of deformable objects**
- D) Software and hardware integration**

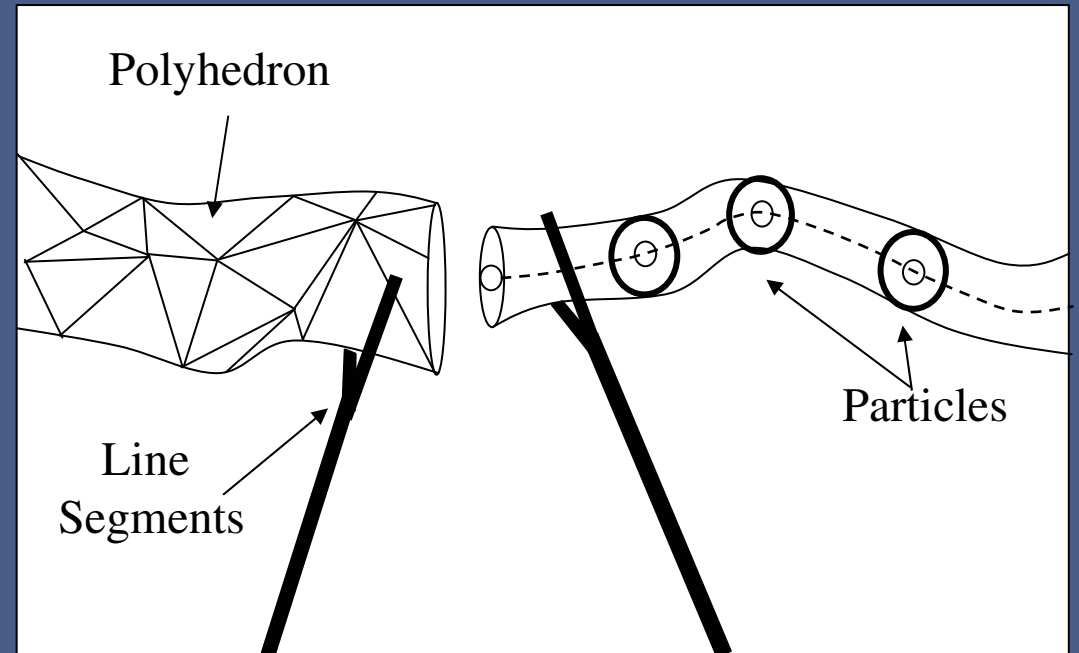
# Case Study:



## Simulation of Catheter Insertion into the Cystic Duct



What you see ...

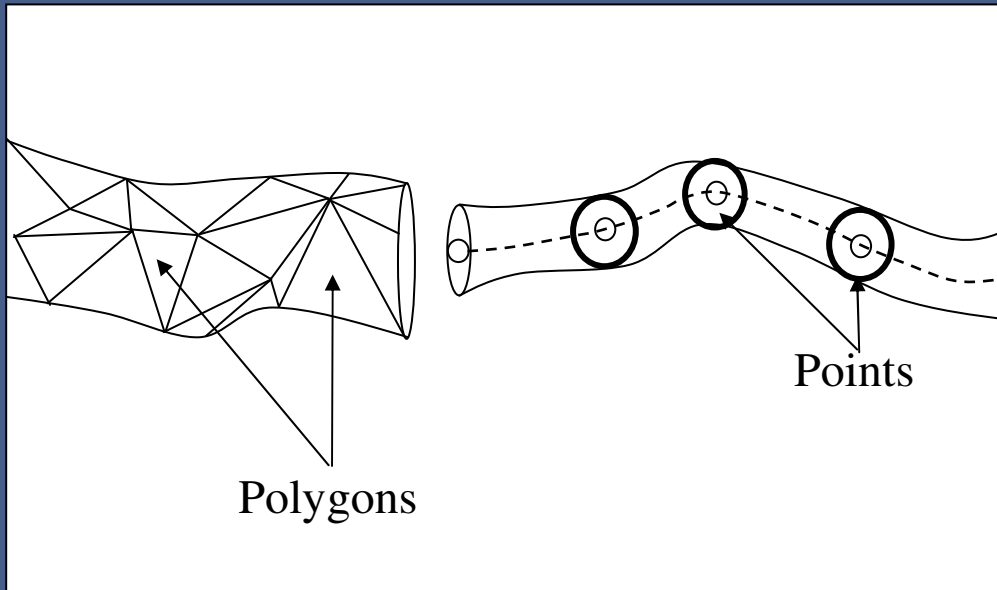


What is really happening ...

# Principles of Collision Detection

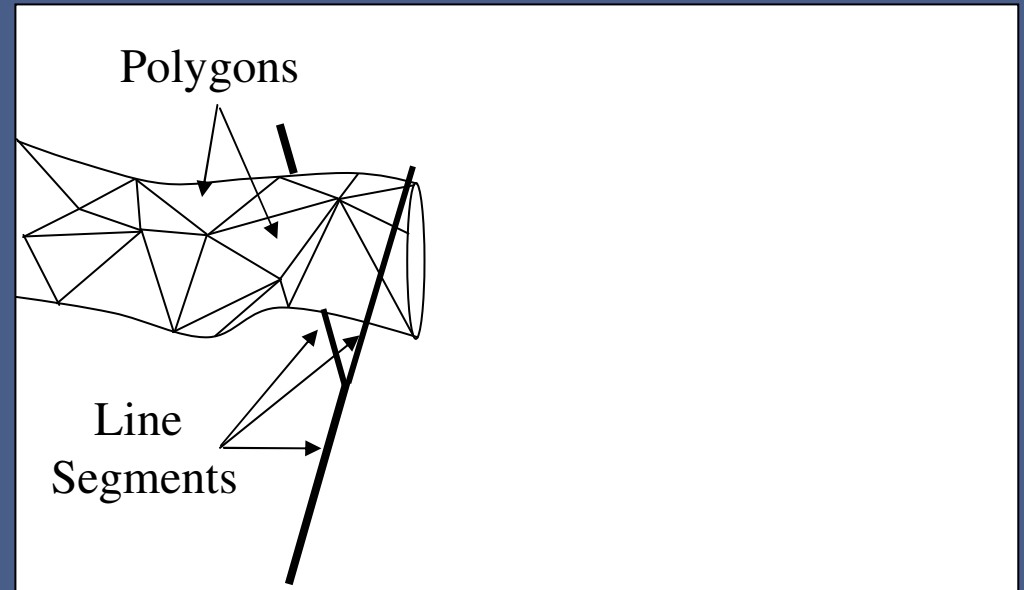
(object-object is too expensive !)

**Point-Object**



(e.g. catheter - cystic duct)

**Line Segment - Object**

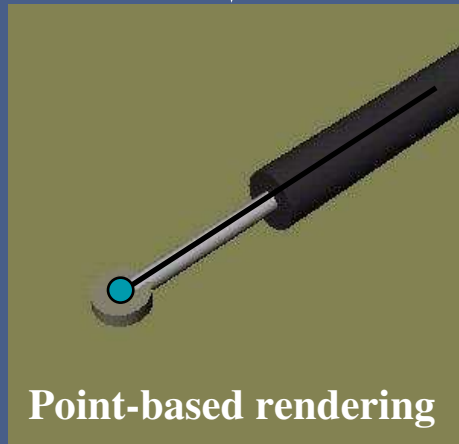


(e.g. forceps - cystic duct)



# Computational Models of Laparoscopic Instruments

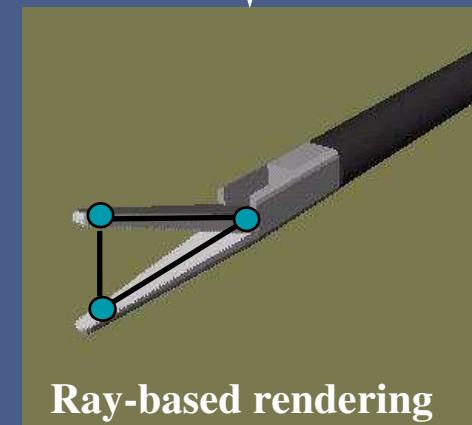
Group A



Point-based rendering



Group B



Ray-based rendering



# Physically-based modeling for simulating soft tissue behavior

---

- Desired properties of deformable models
- Modeling of deformable objects
  - 1) particle-based
  - 2) FEM-based
- Implementing constraints
- Problems with particle-based techniques
- Problems with FEM techniques

# Desired properties of force-reflecting deformable models

---

- reflect stable forces
- display smooth deformations
- handle various boundary conditions and constraints
- display “physically-based” behavior in real-time

# Modeling of Deformable Objects

visit my web-site for the details : <http://eis.jpl.nasa.gov/~basdogan>

## Particle-based :

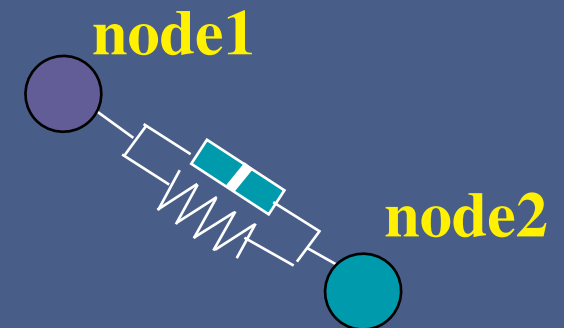
$$F = ma$$

$$\begin{cases} \text{---} & F_{\text{spring}} \\ \text{---} & F_{\text{damping}} \\ \text{---} & F_{\text{gravity}} \end{cases}$$

$$a(t + \Delta t) = F/m$$

$$v(t + \Delta t) = v(t) + \Delta t a(t + \Delta t)$$

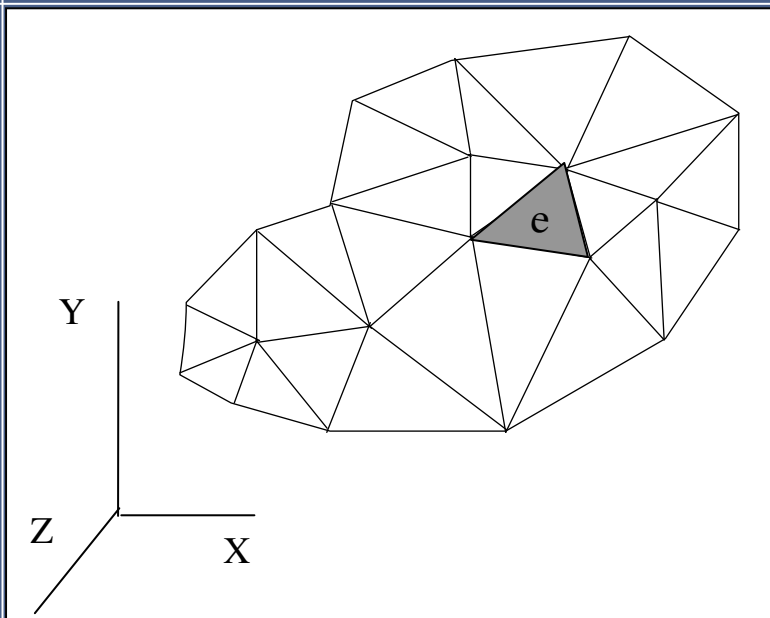
$$p(t + \Delta t) = p(t) + \Delta t v(t + \Delta t)$$



## FEM-based :

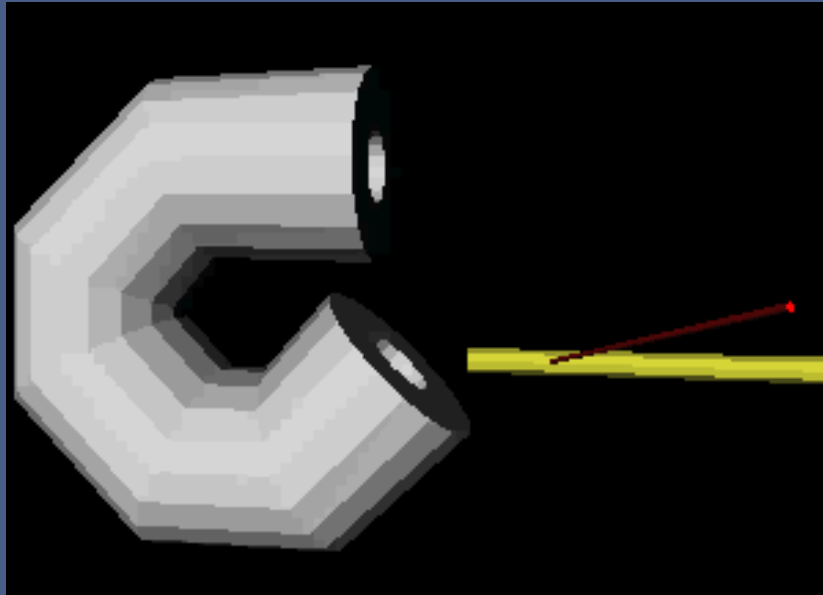
$$F = KU \quad \text{(static analysis)}$$

$$F = MU + CU + KU \quad \text{(dynamic analysis)}$$



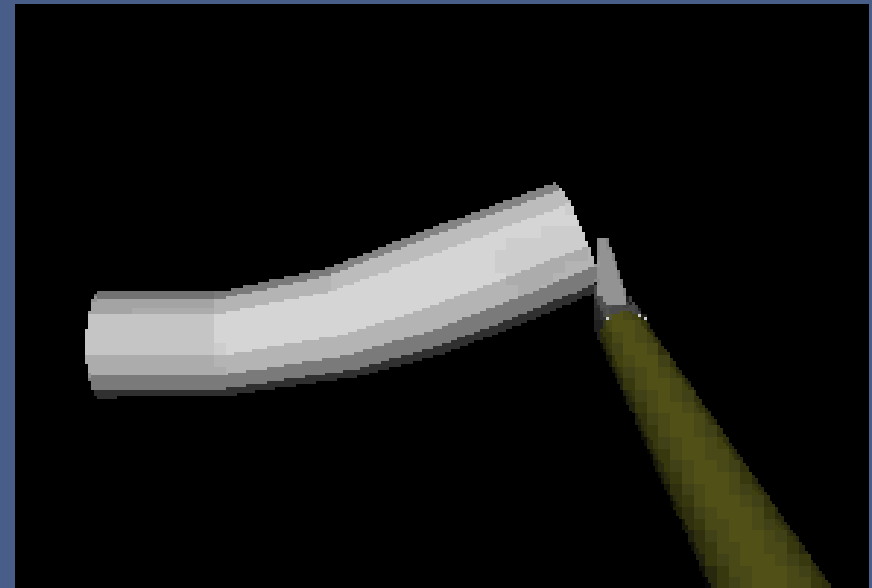
# Comparison

**Particle-based**

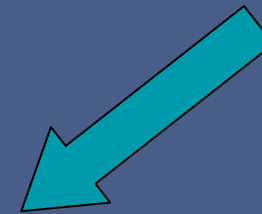
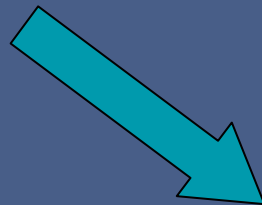


easy to implement, flexible

**FEM-based**



comprehensive



**Hybrid Approach**

# Constraints

(see my SIGGRAPH'99 Course Notes for details, <http://eis.jpl.nasa.gov/~basdogan>)

---

## Examples:

- a node is fixed in 3D space
- a node is constrained to stay on a path
- curvature constraint
- constant volume

## Implementation:

- 1) Particle-based models (Ref: Witkin/Baraff, SIGGRAPH'97 Notes)
  - a) Penalty
  - b) Lagrange multipliers
- 2) FEM

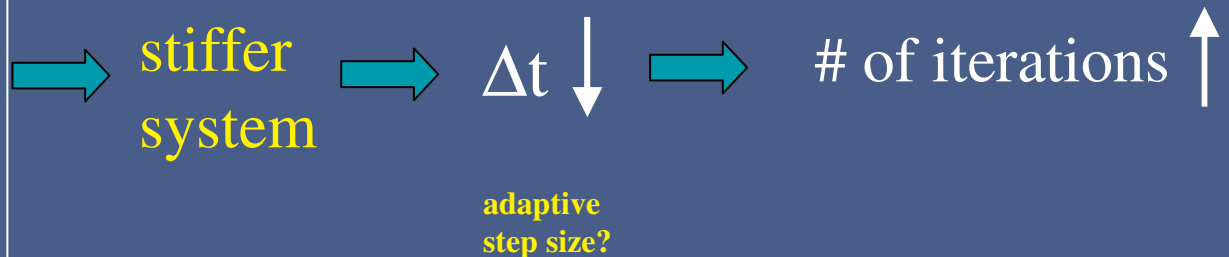
# Problems with Particle-Based Techniques

---

A) Adding damping to stabilize oscillations

B) Adding constraints

C) Too many elements







D) Too few elements → difficult to preserve volume

E) Non-homogeneous distribution of elements → finer adjustment of spring and damper coefficients

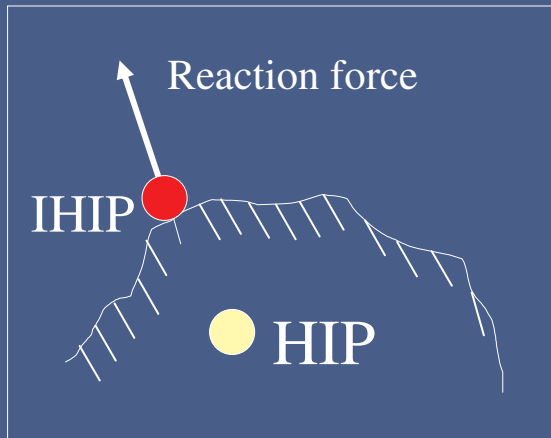
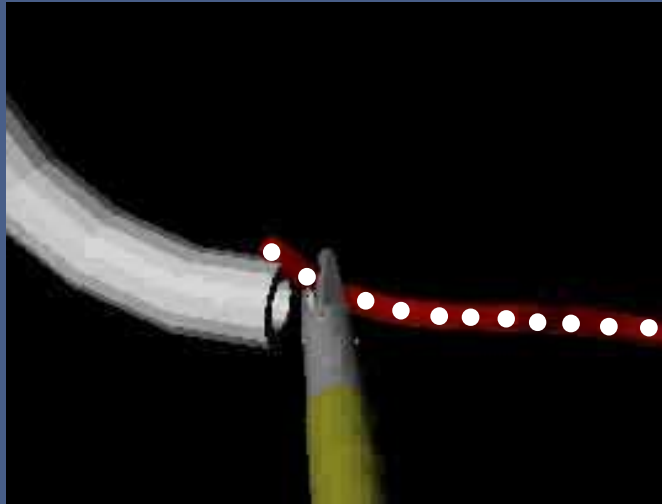
## Problems with FEM Techniques

---

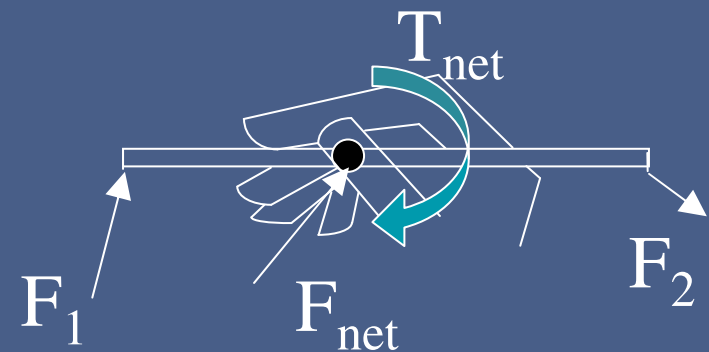
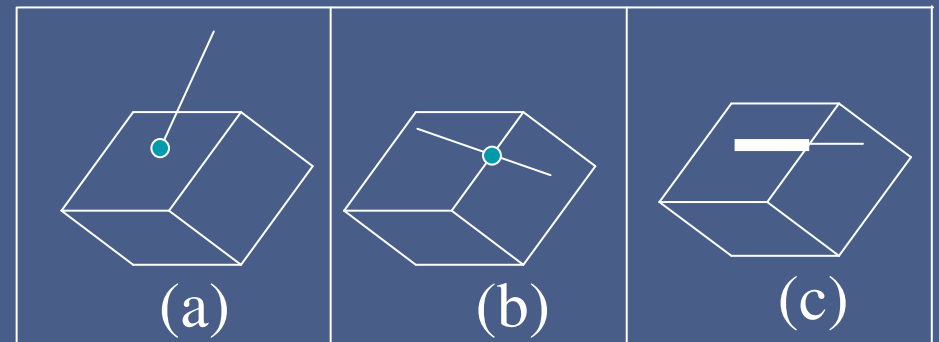
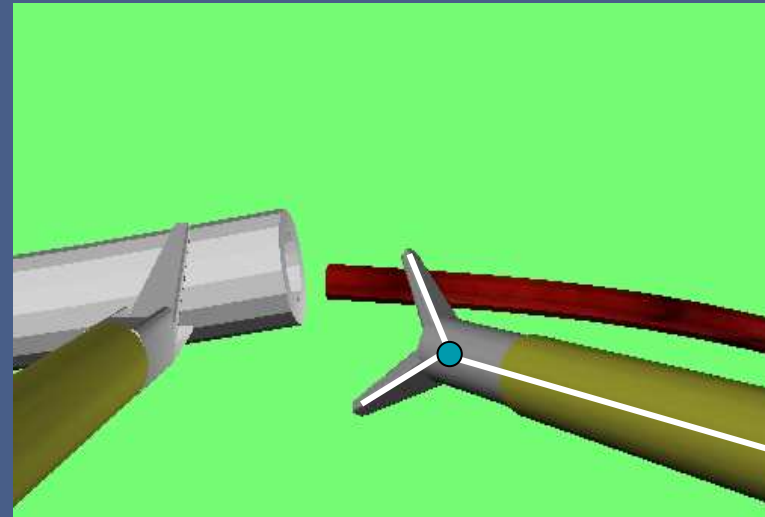
		requires	
A)	Change in topology		Re-meshing
B)	Dynamic analysis		Modeling approximations
C)	Matrix inversion		Pre-computation
D)	Memory allocation		Simplifications in the geometry



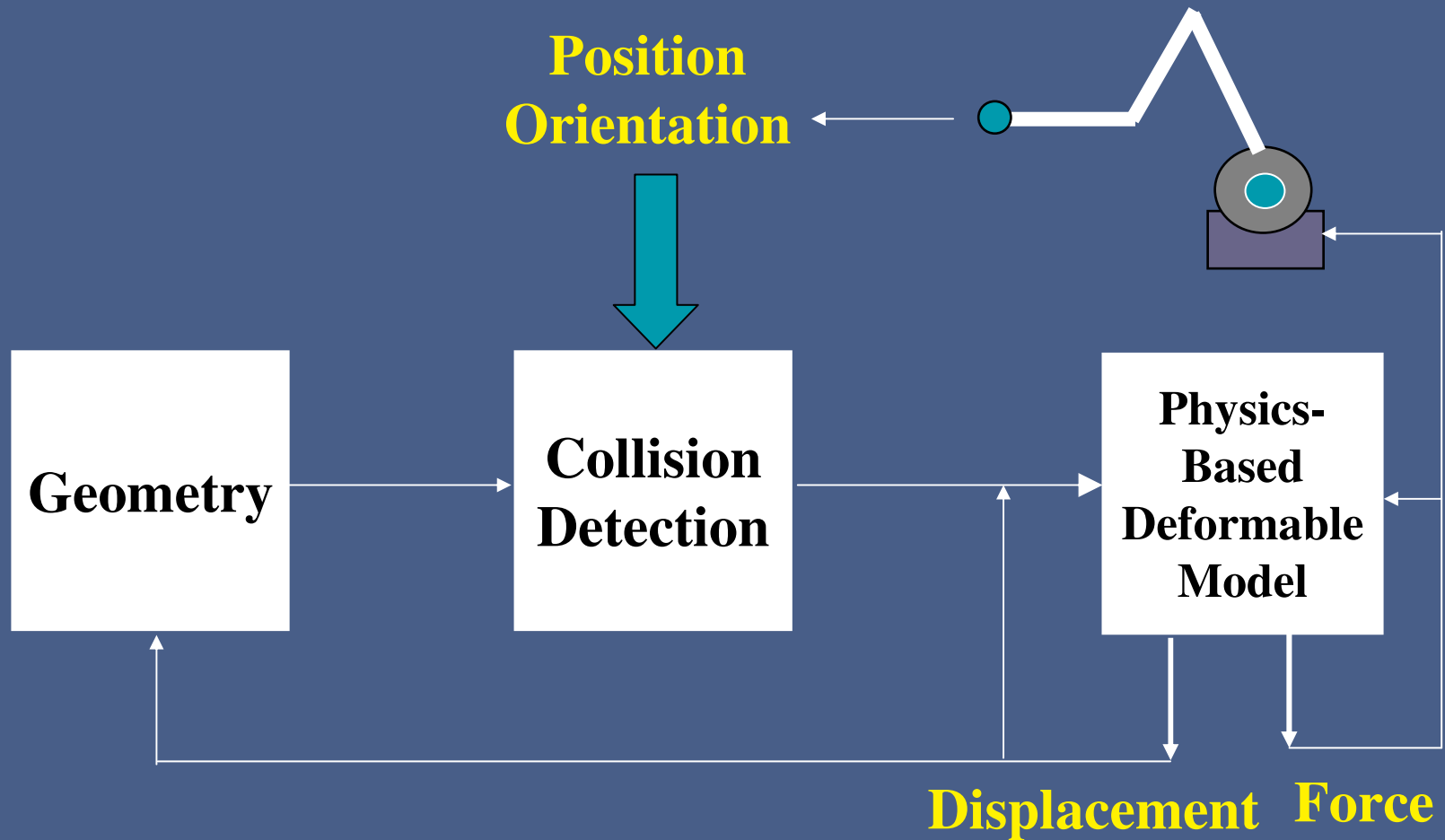
# Point-based



# Ray-based



# Computational Architecture



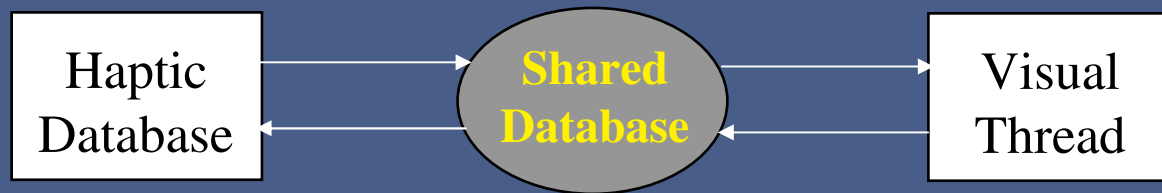
## **D) Software and Hardware Integration: tips and tricks**

---

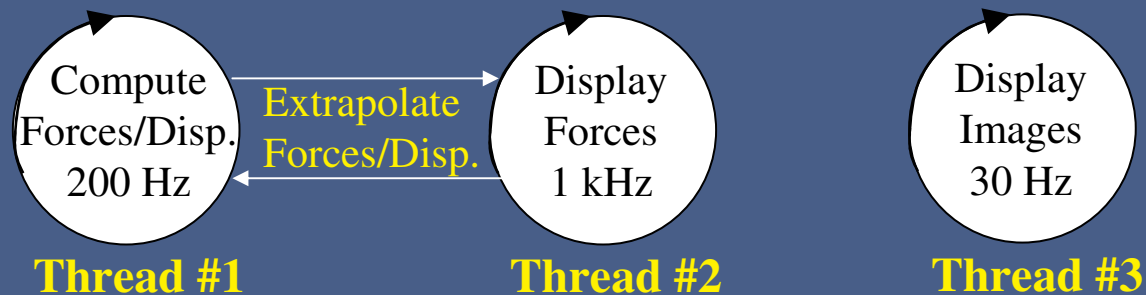
- **Programming tips to speed up your computations**
- **Modeling tips to speed up your computations**
- **Simulation set-up**

# Programming tips to speed up your computations

- Synchronize your haptic and graphic loops through a shared database



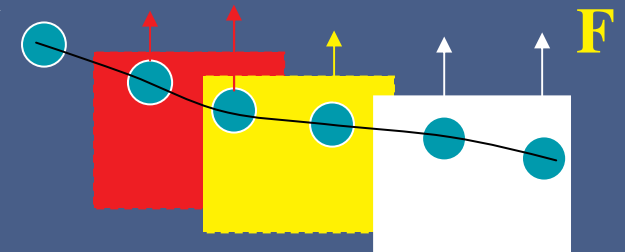
- Construct a multi-layered computing structure if possible



- Construct a hierarchical data structure
- Update your geometric coordinates less frequently

$$\Delta t_{\text{haptic}} = 0.001 \text{ sec (display forces)}$$

$$\Delta t_{\text{iteration}} = 0.01 \text{ sec (update coordinates)}$$



# Modeling tips to speed up your computations

---

- consider local deformation
- take advantage of single point interactions
- condense your matrices in FEM
- consider modeling approximations for dynamic FEM analysis
- pre-compute (matrices, unit displacements/force)
- loosely couple your force and deformation model
- take advantage of human perceptual limitations

## local deformation

```
r = |vertex[i].coord - Collision Point|;
if ( r < Rdeformation )
    vertex[i].frozen = yes;
```

## point-based interactions

$$U = K_i^{-1} F_i$$

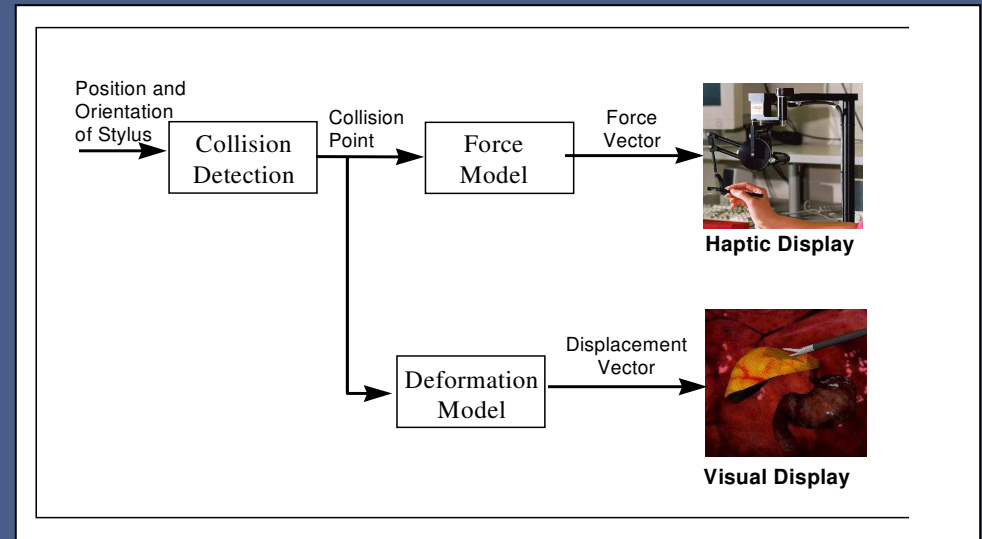
Multiply “i” is the i-th column of  $K^{-1}$  matrix and i-th entry of force vector

## condensation

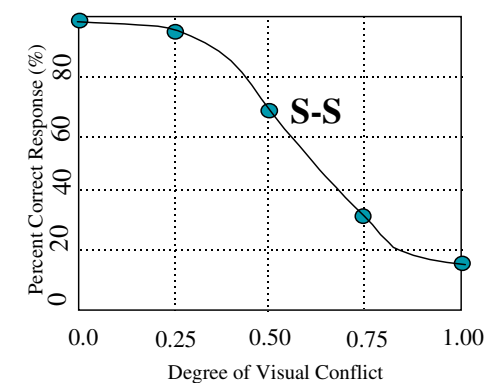
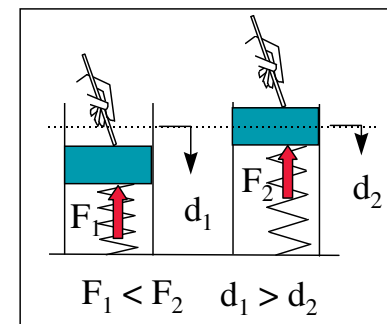
$$\begin{bmatrix} K_{MM} & K_{MS} \\ K_{SM} & K_{SS} \end{bmatrix} \begin{bmatrix} U_M \\ U_S \end{bmatrix} = \begin{bmatrix} F_M \\ F_S \end{bmatrix}$$

- eliminate internal elements
- eliminate rotational dofs

## loose coupling of force and displacement models



## human perceptual limitations



Modeling approximations:

# Modal Analysis

$$F = M\ddot{U} + C\dot{U} + KU \quad (\text{global coordinates, } N \text{ COUPLED equations !})$$

$$U = \Phi X$$

$$f_i = \ddot{X}_i + \alpha_i \dot{X}_i + \omega_i^2 X_i \quad i = 1, \dots, N \quad (\text{modal coordinates, DECOUPLED !})$$

modal reduction  $(R \ll N)$   
 solve for  $X^R, \dot{X}^R, \ddot{X}^R$

$$U, \dot{U}, \ddot{U}$$

Compute F



# Spectral Lanczos Decomposition

$$E'(t) = F_o \frac{1}{K'} (1 - e^{-\frac{\alpha}{2} t} \cos(\sqrt{K' - (\alpha^2 / 4)} I t)) + \frac{\alpha}{2} \frac{1}{\sqrt{K' - (\alpha^2 / 4)} I} e^{-\frac{\alpha}{2} t} \sin(\sqrt{K' - (\alpha^2 / 4)} I t)$$

**K'** (not diagonal !)  
N x N



Lanczos Decomposition

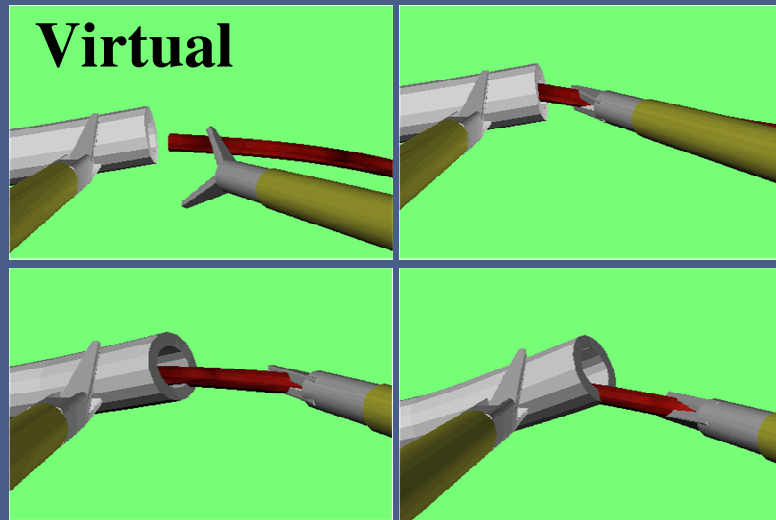
$$E'(t) = F_o QV \left[ \frac{1}{\Lambda} (1 - e^{-\frac{\alpha}{2} t} \cos(\sqrt{\Lambda - (\alpha^2 / 4)} t)) + \frac{\alpha}{2} \frac{1}{\sqrt{\Lambda - (\alpha^2 / 4)}} e^{-\frac{\alpha}{2} t} \sin(\sqrt{\Lambda - (\alpha^2 / 4)} t) \right] V^T e_1$$

**Λ** (diagonal !)  
M x M

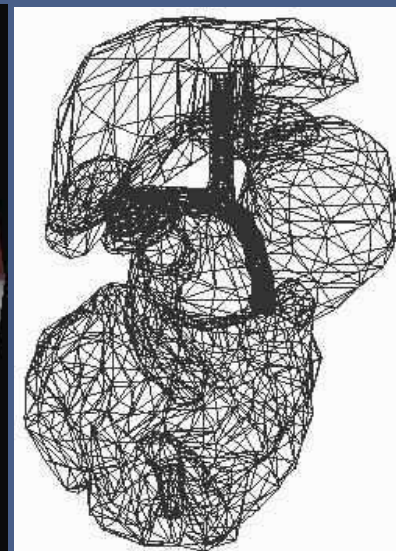
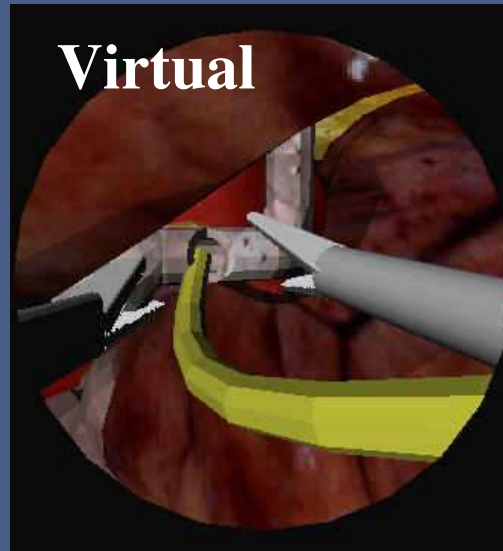
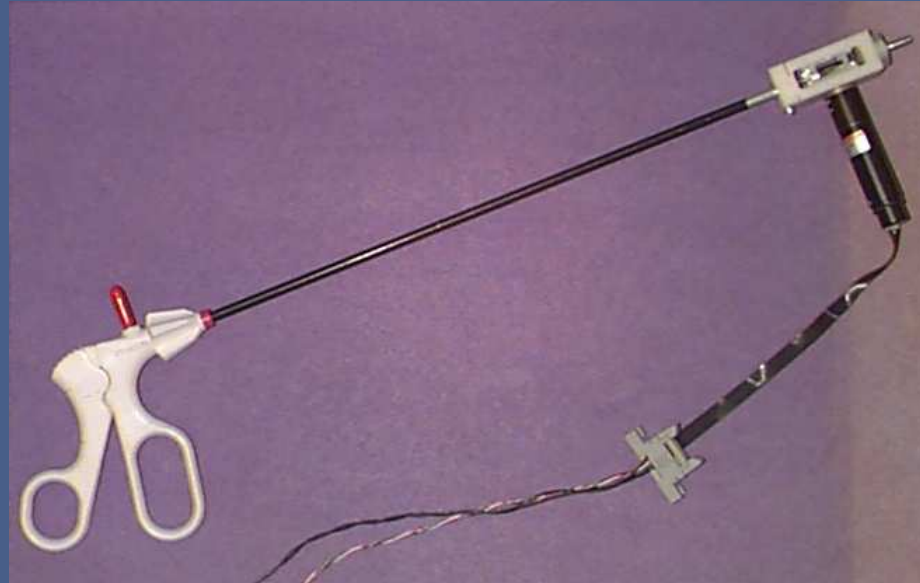
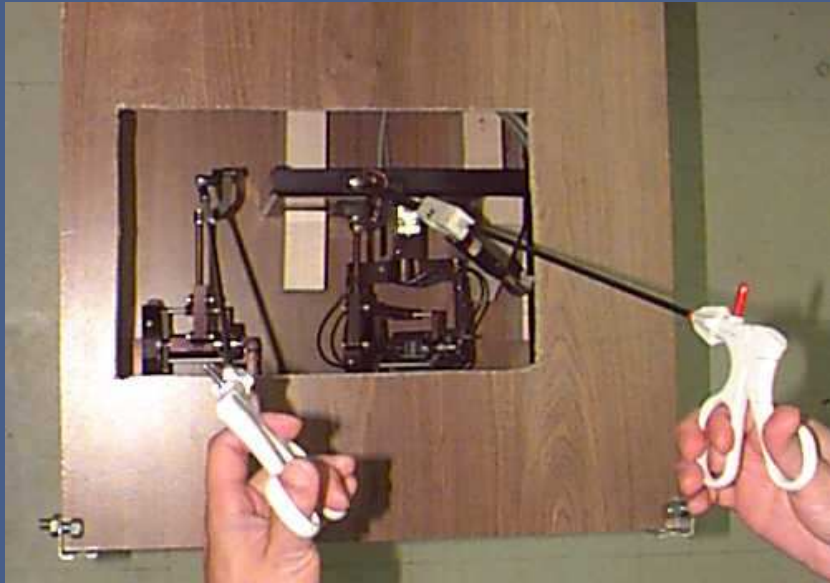
(M << N)



# Simulation of Catheter Insertion



Simulation Set-Up



Geometric Model

tutorial notes are available online:  
**Siggraph'99, MMVR'00, MMVR'01**

<http://network.ku.edu.tr/~cbasdogan>

## **Acknowledgement:**

- haptic rendering  
Chih-Hao Ho, Mandayam Srinivasan, MIT
- design of force reflecting grippers  
Ela Ben-Ur, Mark Ottensmeyer, Ken Salisbury, MIT
- discussions on finite element modeling  
Suvranu De, MIT
- discussions on medical procedures, videos, and several OR visits  
Steve Small, Steve Dawson, David Rattner, MGH-Harvard Medical School
- laparoscopic trainer, several visits to animal facility  
Cynthia Barlow (Harvard Medical School)
- set-up  
Chih-Hao Ho, MIT