Introduction to Robotics

Assignment #4

Due: 01.06.2021, 23:59

Task 4.1 (9 points) Jacobian and singularities: Figure 1 shows a 2-joint planar manipulator with the following constraints: $10° \leq \theta_1 \leq 350°$, $0° < \theta_2 < 360°$ and $l_1 > l_2$.

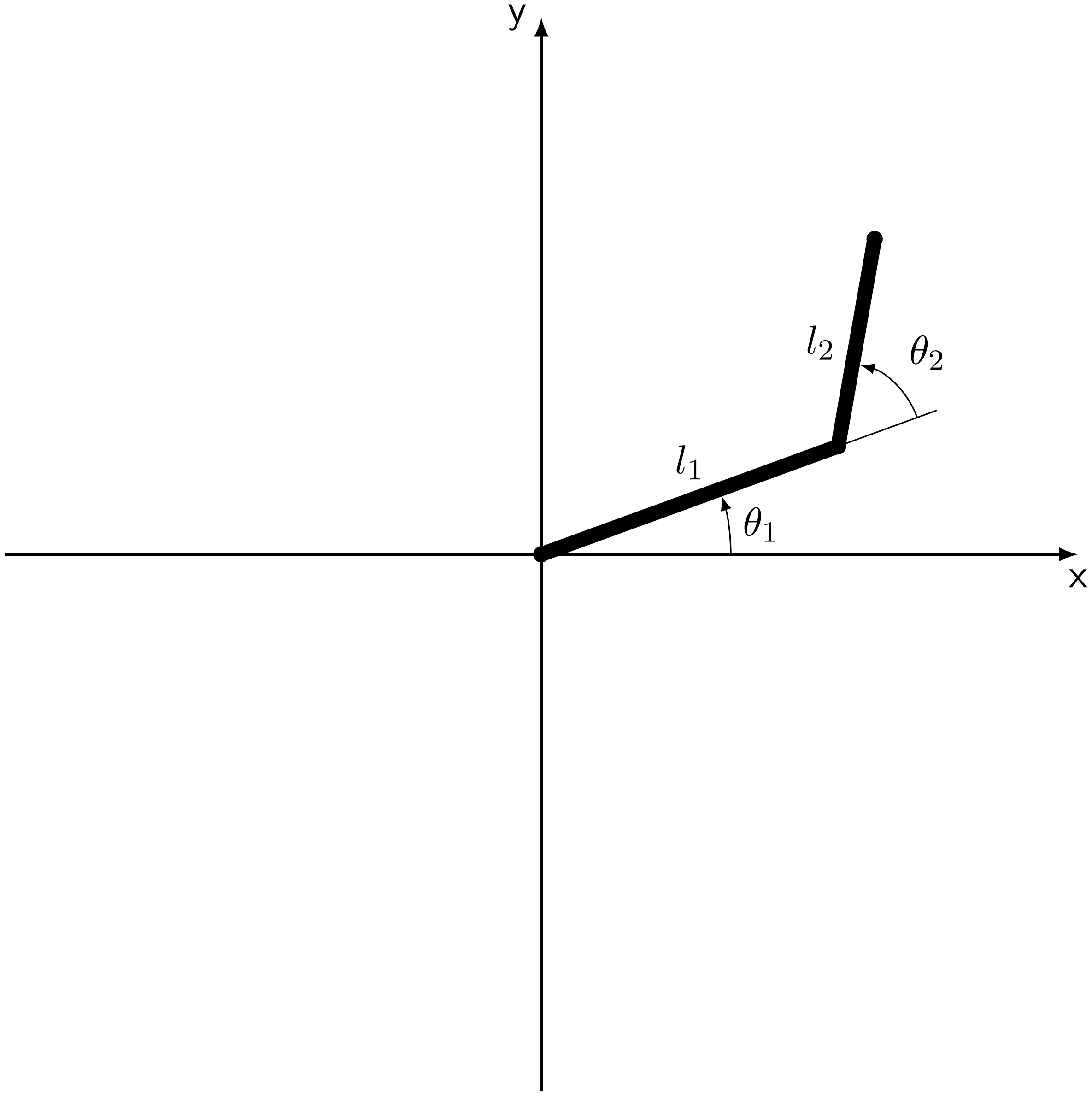

Figure 1: 2-joint planar manipulator.

4.1.1 (2 points): Illustrate the workspace of the manipulator.

4.1.2 (3 points): Determine the Jacobian matrix for the manipulator.

4.1.3 (2 points): Determine the singular configurations of the manipulator (mathematically or geometrical).

4.1.4 (2 points): Outline and explain the determined singular configurations (mathematically or geometrical).

Task 4.2 (2 points) Jacobian: Extend the Jacobian matrix for the 3-joint planar manipulator shown in figure 2.

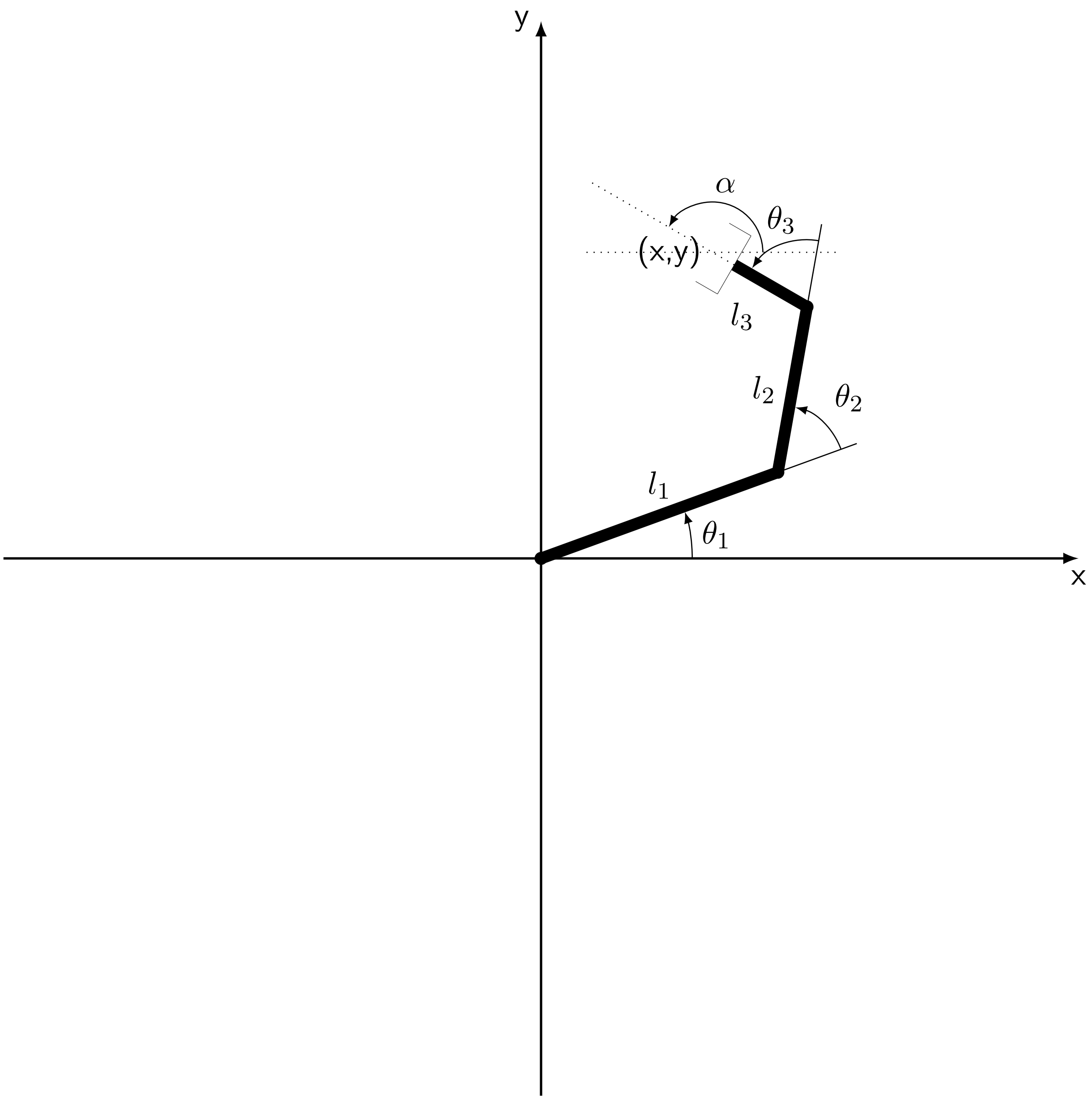

Figure 2: 3-joint planar manipulator.

Task 4.3 (4 points) Singularities of a PUMA560: Consider a PUMA560 manipulator as shown in figure 3. Explain at least three of the possible singular configurations!

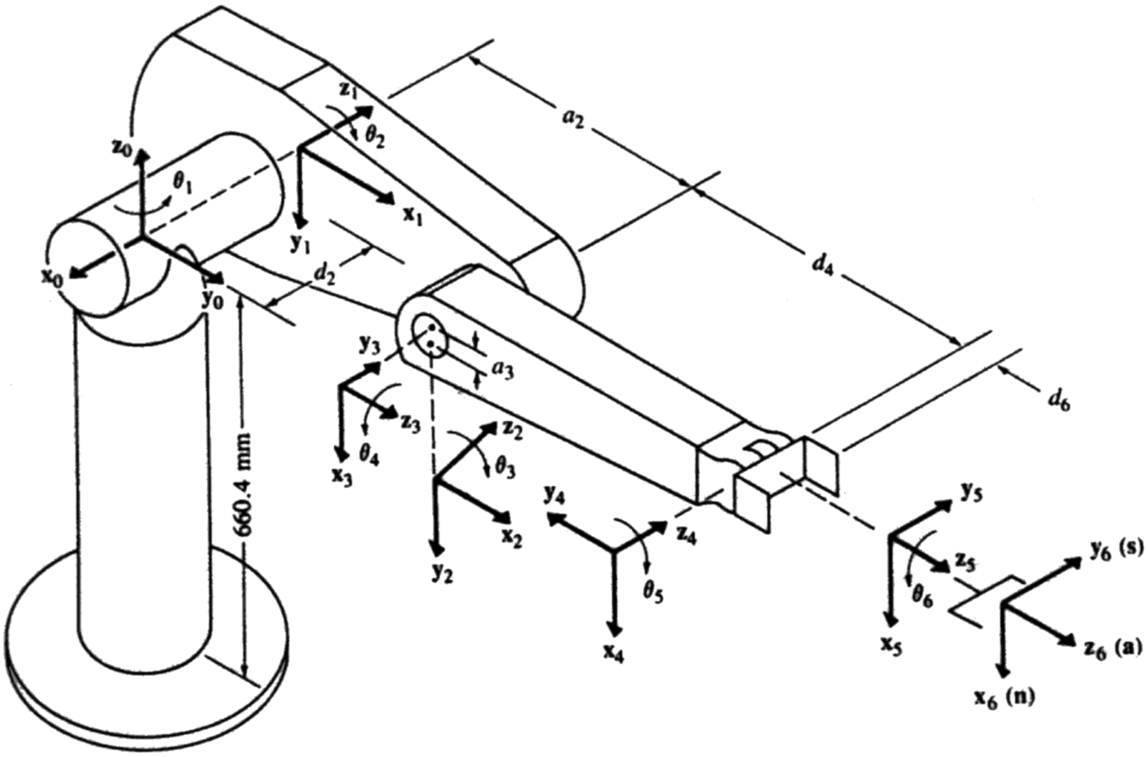

Figure 3: PUMA560 manipulator.

Hint: `Workspace boundary singularities` occur whenever the manipulator is fully extended or is folding back onto itself.

`Workspace-internal singularities` occur if two or more joint axes enter a collinear configuration.

Task 4.4 (5 points) Homogeneous transformation: The transformation of the rotation with θ around an arbitrary vector $\boldsymbol{k} = k_x \overrightarrow{i} + k_y \overrightarrow{j} + k_z \overrightarrow{k}$ ($\overrightarrow{i}$, $\overrightarrow{j}$, $\overrightarrow{k}$ are three unit vectors coinciding with x, y, z) is defined as:

$$Rot_{\mathbf{k},\theta} = \begin{bmatrix} k_x k_x V\theta + C\theta & k_y k_x V\theta - k_z S\theta & k_z k_x V\theta + k_y S\theta & 0 \\ k_x k_y V\theta + k_z S\theta & k_y k_y V\theta + C\theta & k_z k_y V\theta - k_x S\theta & 0 \\ k_x k_z V\theta - k_y S\theta & k_y k_z V\theta + k_x S\theta & k_z k_z V\theta + C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where $C\theta = \cos\theta$, $S\theta = \sin\theta$
and $V\theta = \text{versine}\,\theta = 2\sin^2(\frac{\theta}{2}) = 1 - \cos\theta$,

Derive the homogenous transformation $Rot_{\mathbf{k},\theta}$ in equation (1).

See R. P. Paul, Robot Manipulators: Mathematics, Programming and Control, MIT Press, 1986, section 1.12 "General Rotation Transformation.