Introduction to Robotics

Lecture 08

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Technical Aspects of Multimodal Systems

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Idea

- Planning on graphs of reasonable size is simple
- Operating on grids ignores continuous spaces in $\mathcal{X}_{free}$
- Instead rely on **Probabilistic Sampling** to represent the space

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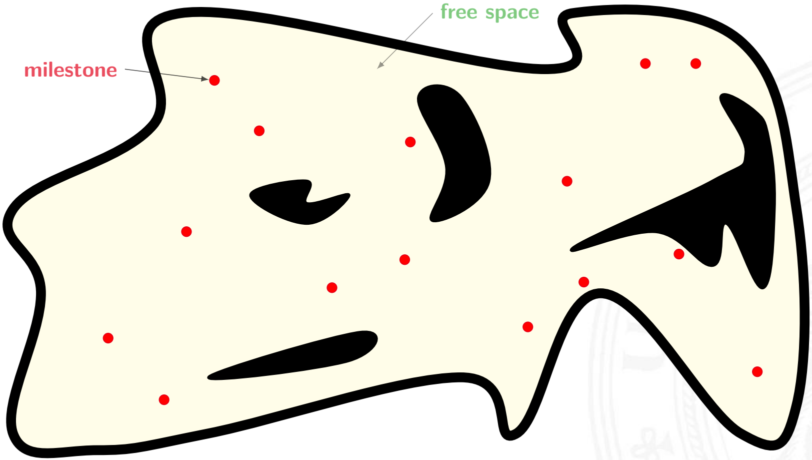
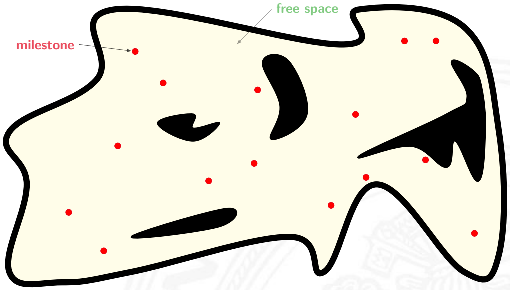


Idea

Key questions:

- How to generate the samples?
- How can the samples be connected to form a planning graph?
- How many samples do you need to describe the space?



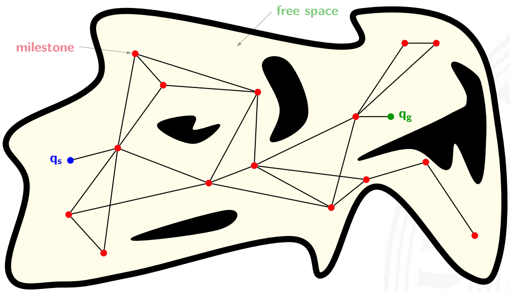
Abstract C-space with sampled valid states

Probabilistic Road Maps

Proposed by Lydia E. Kavraki et.al. 1996 [17]

Two Step algorithm:

1. Construction Phase - Build Roadmap
2. Query Phase - Connect start and goal to graph and solve graph search



Abstract C-space with sampled valid states

Probabilistic Road Maps - Algorithm

Algorithm: sPRM

$V \leftarrow \{x_{\text{init}}\} \cup \{\texttt{SampleFree}_i\}_{i=1,\ldots,n};\ E \leftarrow \emptyset;$

foreach $v \in V$ **do**

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return $G = (V, E);$

Adapted from [15]

Milestones and Roadmap - Construction

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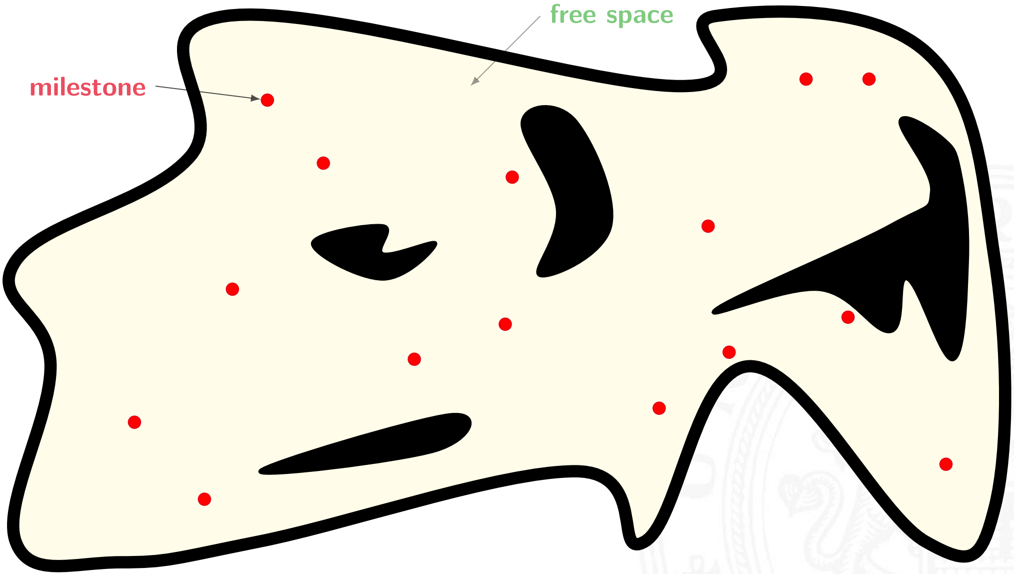

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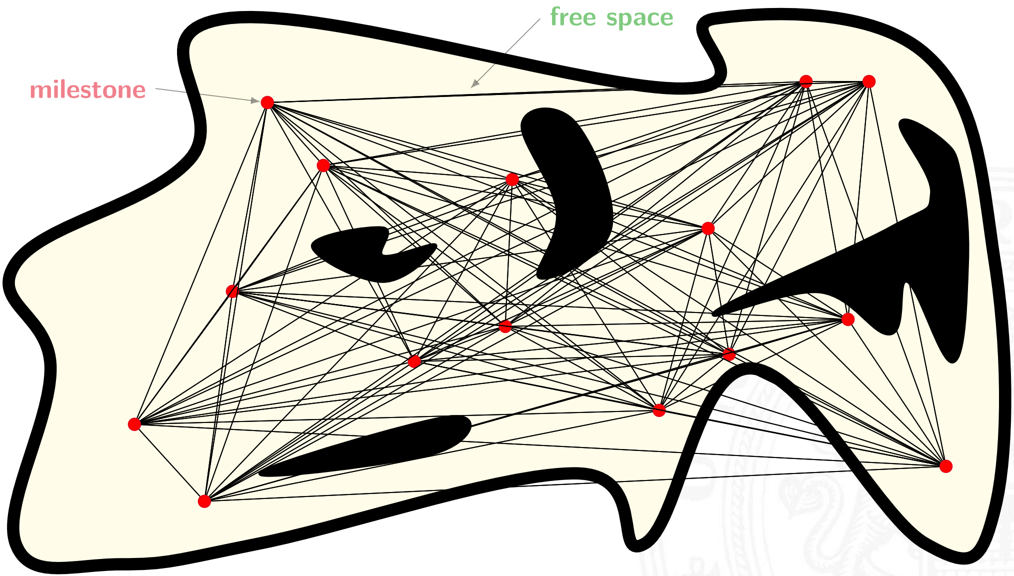

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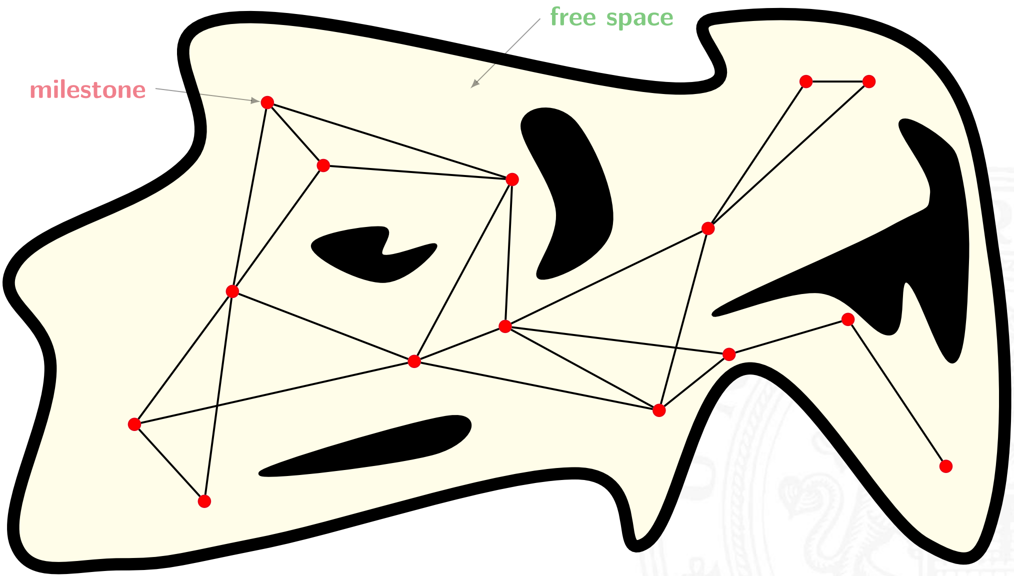
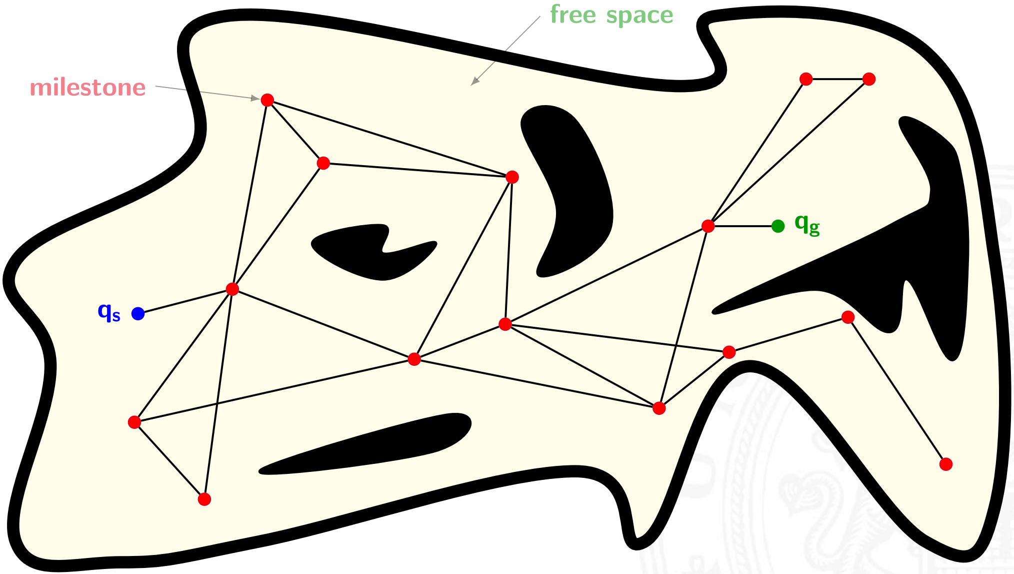


Milestones and Roadmap - Query



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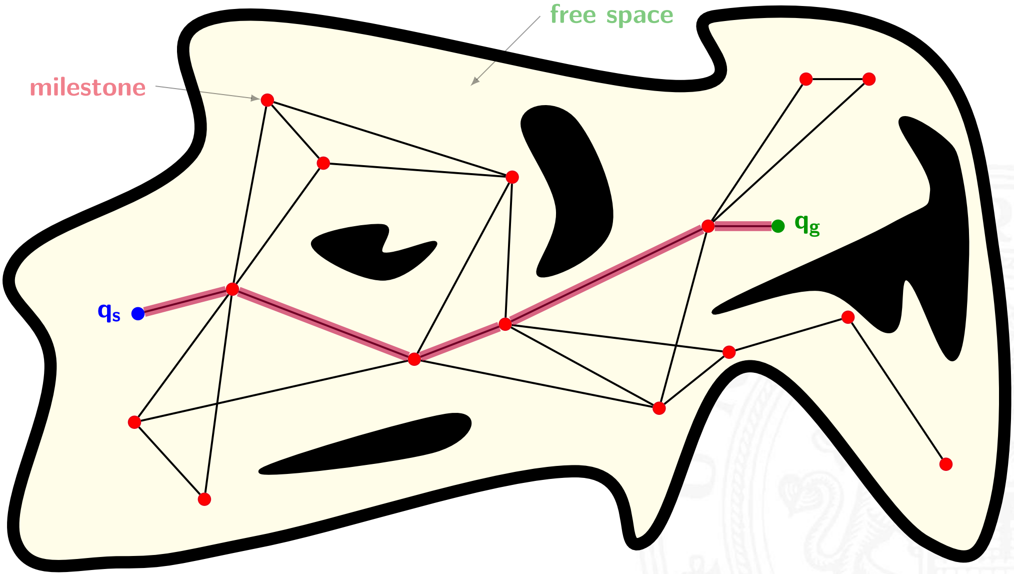

Probabilistic Road Maps - Aspects

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- `SampleFree` - Sample states from $\mathcal{X}_{free}$
- `Near` - Choose Distance metric and threshold
- `CollisionFree(v,u)` - Check motion **between** states for collisions

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`SampleFree` – sample states from $\mathcal{X}_{free}$

- Traditionally: Rejection Sampling
  Take samples uniformally, add sample if $x \in \mathcal{X}_{free}$
- Alternatives:
  - Projective Sampling: Replace samples $x \in \mathcal{X}_{obs}$ by closest state $x' \in \mathcal{X}_{free}$
  - Generative Sampling: For a sufficient **parameterized** space $\mathcal{X}'_{free} \subseteq \mathcal{X}_{free}$:
    Sample from $\mathcal{X}'_{free}$ via parameters

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Near - choose distance metric and threshold

- Traditional C-space metric: L_1 distance
- Obvious alternatives: weighted L_1 distance, L_2 distance

- Higher threshold: more negative collision checks
- Lower threshold: slower graph building

Probabilistic Road Maps - Aspects

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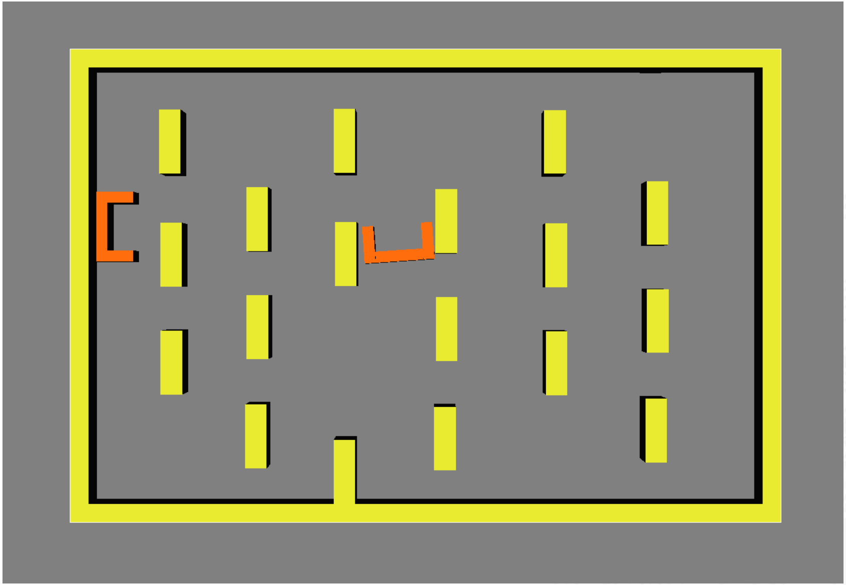
`CollisionFree(v, u)` - Local Planning

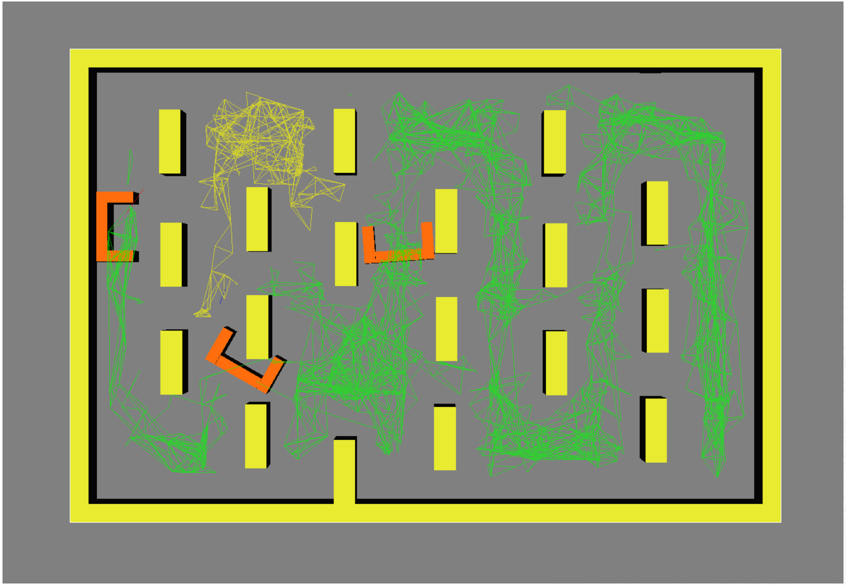
- Traditionally collision-checking tests **one** state
- Interpolate states between $\langle v, u \rangle$ and check those
  - Fixed step size in C-space can imply huge motions in workspace!
- Continuous collision checking (CCD):
  - Current systems rely on primitive motions
  - Robot links move in complex splines

Example



3dof planning problem

Example



3dof planning problem

Single and Multi-Query Planning

Definition

If only a single path is requested in a potentially changing scene, this is called **single-query** planning. If datastructures remain valid between motion requests, this is called **multi-query** planning.

PRM solves a multi-query problem by building an undirected graph.

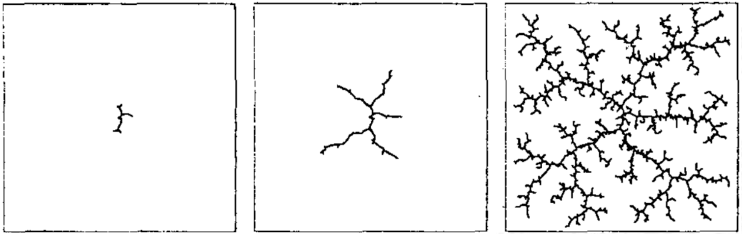
For single-shot planning, the graph search can be avoided altogether.

Rapidly-exploring Random Trees (RRT) - Basic Idea

Proposed by Kuffner and LaValle 2000 [18]

Instead of building a graph, grow a tree from the start state.

If for any leaf state $x \in \mathcal{X}_{goal}$, a solution is found.



RRT at multiple stages of extension

Rapidly-exploring Random Trees (RRT) - Algorithm

Algorithm 3: RRT

$V \leftarrow \{x_{\text{init}}\};\ E \leftarrow \emptyset;$
for $i = 1, \ldots, n$ **do**
    $x_{\text{rand}} \leftarrow \texttt{SampleFree}_i;$
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    if $\texttt{ObtacleFree}(x_{\text{nearest}}, x_{\text{new}})$ **then**
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`Steer(x,y)` - Compute new state x'

- Move from x towards y: $\|y - x'\| < \|y - x\|$
- $\|x - x'\| < \eta$ to limit step size
- Alternatively compute closest $x' \in \mathcal{X}_{free}$ reachable via straight motion

Adapted from [15]

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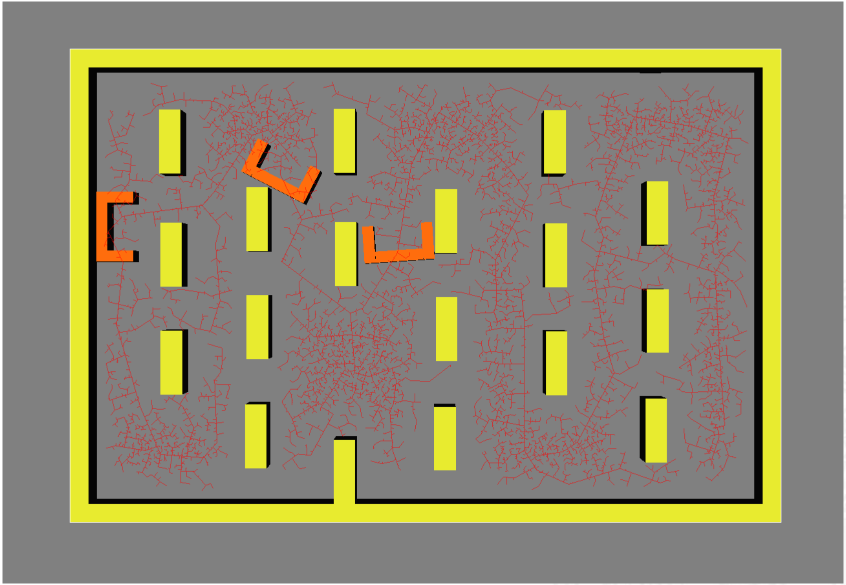
`SampleFree` – sample states from $\mathcal{X}_{free}$

- Traditionally: uniform sampling
- To improve heuristically, a **Goal Bias** can be added
  - Low fraction of samples are sampled from $\mathcal{X}_{goal}$
  - Required if $\mathcal{X}_{goal}$ is small in $\mathcal{X}$

Adapted from [15]

Example



RRT graph of an example

Bi-Directional Search

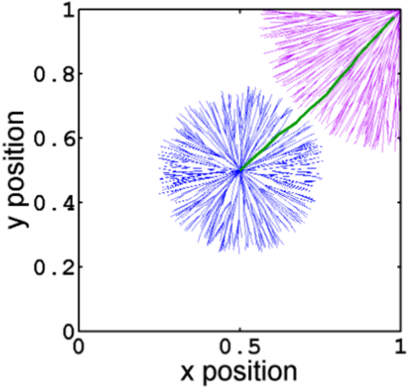
In robotics, start and goal are often in constraint areas of $\mathcal{X}_{free}$, e.g., close to obstacles.

The **transition phase** between these states is often quite flexible.

Instead of growing a single tree towards the goal

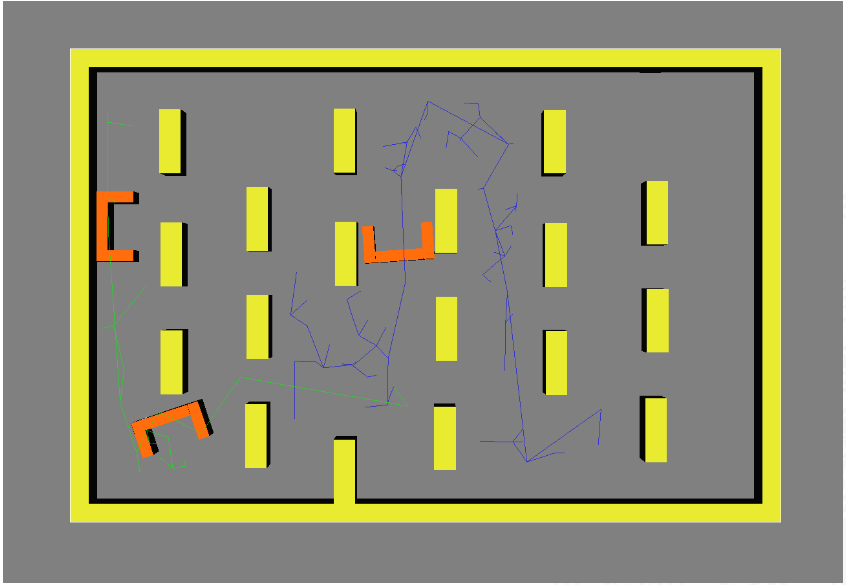
- Grow two trees from start and goal each.
- Attempt to connect them at each step.

In practice, this speeds up planning to the first solution significantly.



Bi-directional search trees [19]

RRT-Connect - Example



RRT-Connect for an example

Expansive Space Trees

PRM and RRT sample random configurations from $\mathcal{X}_{free}$.
Thus they also sample in areas which are already well-represented by milestones.

Definition

The *density* around a state x can be represented by the cardinality of its neighborhood within a distance d: $|N_d(x)|$.

Ideas

- Sample next expansion step weighted by inverse density $w(x) = \frac{1}{|N_d(x)|}$
- Stochastically reject samples in high-density areas

Hsu et.al. 1997 [20]

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Expansive Space Trees - Algorithm

Algorithm *expansion*

1. Pick a node x from V with probability $1/w(x)$.
2. Sample K points from $N_d(x) = \{q \in \mathcal{C} \mid dist_c(q, x) < d\}$, where $dist_c$ is some distance metric of $\mathcal{C}$. (K and d are parameters.)
3. **for** each configuration y that has been picked **do**
4. calculate $w(y)$ and retain y with probability $1/w(y)$.
5. **if** y is retained, $clearance(y) > 0$ and $link(x, y)$ returns YES
6. **then** put y in V and place an edge between x and y.

- Expand from an existing node instead of global samples from $\mathcal{X}$
- Samples rejected in 4. are never collision checked!
- Original formulation is bidirectional

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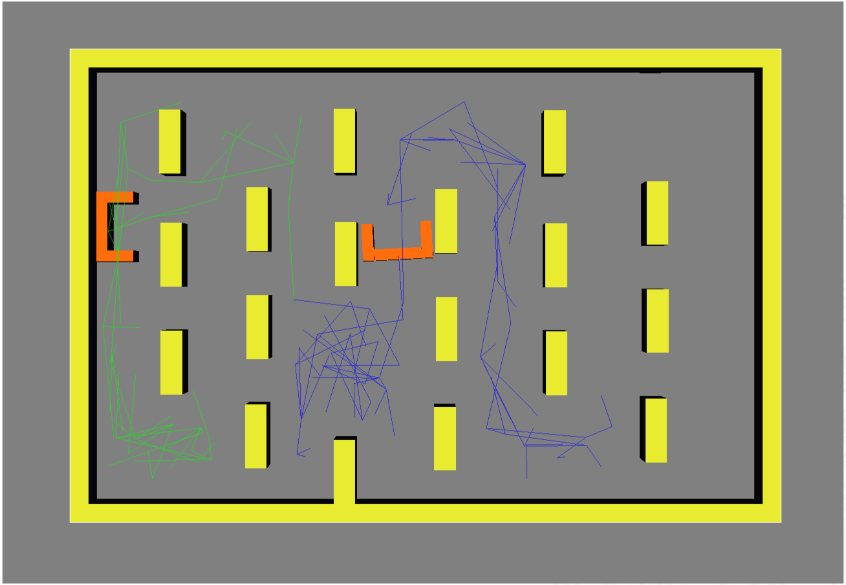
(Bi)EST - Example



(Bi-directional) EST for an example

Path Postprocessing

The resulting paths are not smooth and often contain unnecessary motions.

Traditional post-processing includes:

- Path Shortcutting
  - Repeatedly pick two non-consecutive waypoints and attempt to connect them
- Perturbation of individual waypoints
  - Optional
  - Can reduce solution costs
  - Computationally expensive
  - For differentiable costs: exploit gradient
- Fit smooth splines through waypoints

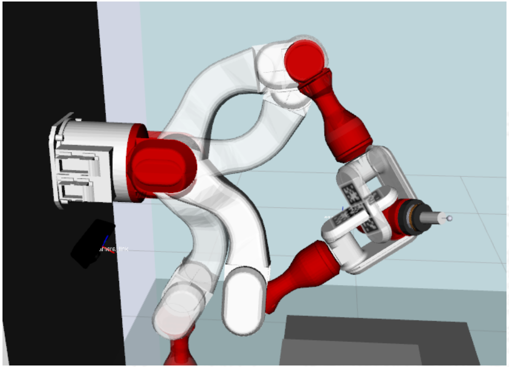
All modifications need to be collision checked.

Multiple IK Solutions

Redundant robots generate multiple joint solutions per pose.

Each Cartesian goal region adds a number of disjoint C-space goal regions.

Most tree-based planners naturally extend to **Multi-Goal Planning**, implicitly building multiple goal trees.



Multiple IK solutions for one target pose © Hendrich

Definition

An **Optimal Path Planning Problem** is defined by a path planning problem $\mathcal{P} = \langle \mathcal{X}_{free}, x_{init}, \mathcal{X}_{goal} \rangle$ and a cost function $c(\tau) : R \geq 0$. It requires to find a feasible path τ^* such that $\tau^* = \text{argmin}_\tau \{ c(\tau) \mid \tau \text{ is feasible for } \mathcal{P} \}$

In practice:

- Two-step process:
  - Find feasible path(s)
  - Optimize path(s)
- Planners are **asymptotically** optimal
  - Convergence might take long
  - Non-trivial to detect ε-optimal solution
- What cost function should be used?
  - C-space path length
  - Accumulated clearance (distance to obstacles)
  - Cartesian end-effector path length
  - Physical work

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Planner*

Method

Instead of stopping at the first trajectory, continue sampling to improve solution.

Karaman and Frazzoli 2011 [15] introduced **PRM*** and **RRT***.
Both are efficient, asymptotically optimal versions of the basic algorithms.

PRM is asymptotically optimal as-is.

- Eventually all points on the optimal path will be added to the roadmap.

Ensure minimal required graph connectivity of $O(n \cdot \log(n))$.

- Reduce the neighborhood radius r with sample size n:

$$r(n) = \gamma_{PRM} \cdot (\frac{log(n)}{n})^{\frac{1}{d}}$$

where γ_{PRM} depends on the planning space, d is the dimensionality of $\mathcal{X}$

Method

Update tree whenever new samples yield cheaper paths to root.

- Instead of connecting the new states to *closest node*, connect to the *cheapest node* in neighborhood
- Change parent of neighboring states to new state if new path is cheaper

RRT* - Algorithm

Algorithm 6: RRT*

$V \leftarrow \{x_{\text{init}}\}$; $E \leftarrow \emptyset$;
for $i = 1, \ldots, n$ **do**
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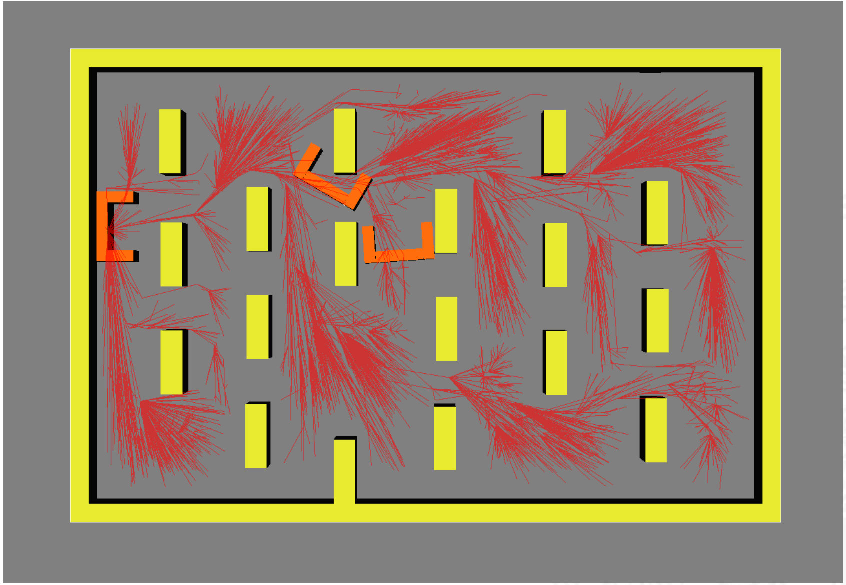
  $x_{\text{new}} \leftarrow \texttt{Steer}(x_{\text{nearest}}, x_{\text{rand}})$;
  if $\texttt{ObtacleFree}(x_{\text{nearest}}, x_{\text{new}})$ **then**
    $X_{\text{near}} \leftarrow \texttt{Near}(G = (V, E), x_{\text{new}}, \min\{\gamma_{\text{RRT}^*}(\log(\text{card}\,(V))/\,\text{card}\,(V))^{1/d}, \eta\})$;
    $V \leftarrow V \cup \{x_{\text{new}}\}$;
    $x_{\text{min}} \leftarrow x_{\text{nearest}}$; $c_{\text{min}} \leftarrow \texttt{Cost}(x_{\text{nearest}}) + c(\texttt{Line}(x_{\text{nearest}}, x_{\text{new}}))$;
    foreach $x_{\text{near}} \in X_{\text{near}}$ **do** // Connect along a minimum-cost path
      if $\texttt{CollisionFree}(x_{\text{near}}, x_{\text{new}}) \wedge \texttt{Cost}(x_{\text{near}}) + c(\texttt{Line}(x_{\text{near}}, x_{\text{new}})) < c_{\text{min}}$ **then**
        $x_{\text{min}} \leftarrow x_{\text{near}}$; $c_{\text{min}} \leftarrow \texttt{Cost}(x_{\text{near}}) + c(\texttt{Line}(x_{\text{near}}, x_{\text{new}}))$
    $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$;
    foreach $x_{\text{near}} \in X_{\text{near}}$ **do** // Rewire the tree
      if $\texttt{CollisionFree}(x_{\text{new}}, x_{\text{near}}) \wedge \texttt{Cost}(x_{\text{new}}) + c(\texttt{Line}(x_{\text{new}}, x_{\text{near}})) < \texttt{Cost}(x_{\text{near}})$
      then $x_{\text{parent}} \leftarrow \texttt{Parent}(x_{\text{near}})$;
      $E \leftarrow (E \setminus \{(x_{\text{parent}}, x_{\text{near}})\}) \cup \{(x_{\text{new}}, x_{\text{near}})\}$
return $G = (V, E)$;

RRT* - Example



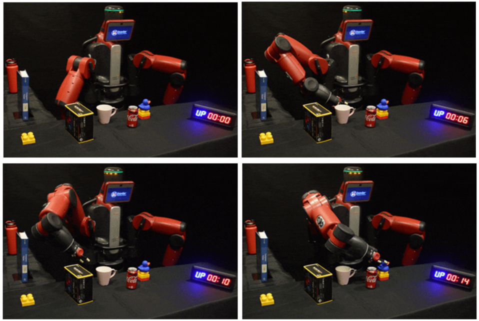
RRT* for an example

Summary - Sampling Based Planning

- Represent $\mathcal{X}_{free}$ probabilistically through samples
- Relies heavily on binary collision checking
- Post-processing solutions is essential
- Various (dozens) of algorithms with varying performance
- Straight-forward extensions for asymptotically optimal planning

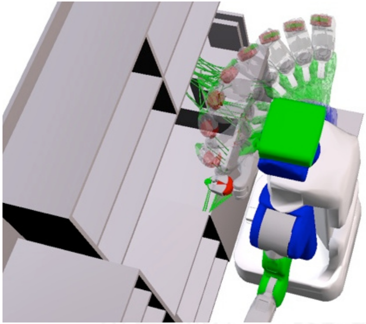
Beyond Sampling-Based Planning

MPNet



Fast deep-learning system learning from planners [21]

TrajOpt



Sequential convex optimizer solving trajectories [22]

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