# Introduction to Robotics 

Lecture 1

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Technical Aspects of Multimodal Systems

April 24, 2020

| Lecture: | Friday 10:15 c.t. - 11:45 c.t. |
| :--- | :--- |
| Room: | F-334 |
| Web: | http://tams.inf...burg.de/lectures/ |
| Exercises | Friday 09:00 c.t. - 11:00 c.t. / |
| /RPC: | Friday $09: 00$ c.t. $-13: 00$ c.t. (alternating) <br>  <br> Ree website for dates |
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- See website for more information

TAMS course website:
http://tams.informatik.uni-hamburg.de/lectures/2020ss/vorlesung/itr

This course is organized with Moodle:
https://lernen.min.uni-hamburg.de/

Lecture

- Intelligent Robotics (winter, Bestmann)
- RoboCup - Playing football with humanoid robots (Summer, Bestmann)
- Lecture Computer Vision I (winter, Frintrop)
- Lecture Computer Vision II (summer, Frintrop)
- Neural Networks (summer, Wermter)

Projects

- Masterproject intelligent robotics (winter, TAMS)
- RoboCup - Playing football with humanoid robots (winter, Bestmann)
- Human-Computer Interaction (winter, Heinecke)


## Previous Knowledge

- Linear algebra
- Essence of linear algebra by 3Blue1Brown
- Basics in physics
- force, torque, work...
- Related computer skills
- Linux (RPC)
- Python (RPC and Excercises)
- Matlab (Excercises)
- git (RPC)
- access to mafiasi.de and pool computers


## Own Hardware

If you use your own laptop, you require a Ubuntu 18.06 (Live or Virtual Machine) and fully installed ros-melodic-desktop-full

PR2 robot


## Content

- Mathematic concepts
- spatial description
- kinematics
- dynamics
- Control concepts
- movement execution
- Programming aspects
- ROS, URDF, Kinematics Simulator
- Task-oriented movement and planning


## Schedule

## Slides \& Dates

| 24.04. | $\# 01$ | $[E X]$ Introduction, Coordinate Systems |
| :--- | :--- | :--- |
| 01.05. | $\# 02$ | $[N O]$ Kinematics, Robot Description |
| 08.05. | $\# 03$ | $[R P C]$ Robot Description, Inverse Kinematics |
| 15.05. | $\# 04$ | $[E X]$ Differential Motion |
|  | $\# 05$ | $[E X]$ Jacobian |
| 22.05. | $\# 06$ | $[R P C]$ Trajectory Planning |
| 29.05. | $\# 07$ | $[E X]$ Trajectory Generation |
| 05.06. | No lecture | (Holiday) |
| 12.06. | $\# 08$ | $[R P C]$ Dynamics |
| 19.06. | $\# 09$ | $[E X]$ Robot Control |
| 26.06. | $\# 10$ | $[R P C]$ Task-oriented Trajectory Generation and Object Representation |
| 03.07. | $\# 11$ | $[E X]$ Path Planning |
| 10.07. | $\# 12$ | $[R P C]$ Architectures of Sensor-Based Intelligent Systems |
|  | $\# L C$ | $[R P C]$ Summary, Conclusion, Outlook |

## Outline

Introduction

> Basic Terms
> Degree of Freedom
> Robot Classification

Spatial Description and Transformations
Forward Kinematics
Robot Description
Inverse Kinematics for Manipulators
Differential motion with homogeneous transformations
Jacobian
Trajectory planning
Trajectory generation
Dynamics
Principles of Walking

## Outline (cont.)

Robot Control
Task-Level Programming and Trajectory Generation
Task-level Programming and Path Planning
Task-level Programming and Path Planning
Architectures of Sensor-based Intelligent Systems
Summary
Conclusion and Outlook

Robot became popular through a stage play by Karel Čapek in 1920, being a capable servant.

Robotics was first used by Isaac Asimov in 1942.
Three Laws of Robotics

1. A robot may not injure a human being or, through inaction, allow a human being to come to harm.
2. A robot must obey the orders given it by human beings except where such orders would conflict with the First Law.
3. A robot must protect its own existence as long as such protection does not conflict with the First or Second Laws.

## Obey or not



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${ }^{1}$ https://irobot.fandom.com/wiki/I,_Robot_(film)
${ }^{2}$ https://www.rottentomatoes.com/tv/westworld/s03

Legged-robots in Boston Dynamics


SpotMini


Spot

Boston Dynamics 2


Atlas

[^0]
## Advanced robots

## Medical Robot



456
${ }^{4}$ https://www.dlr.de/content/en/articles/news/2019/02/20190507_dih-hero-a-medical-roboticsnetwork.html
${ }^{5}$ https://newatlas.com/hyundai-robotic-exoskeleton/43331/
${ }^{6}$ https://www.youtube.com/watch?v=wOzw71j4b78\&t=4s

## Advanced robots

## Industrial Robot


${ }^{7}$ https://www.robotics.org/blog-article.cfm/Industrial-Robot-Sales-Broke-Records-in-2018/136


## Robotics

Intelligent combination of computers, sensors and actuators.


## Degree of Freedom (DOF)

The number of variables to determine position of a control system in space.

- Point on a line
- Point on a plane
- Point in space
- Rigid body
- in space
- on a plane
- Non-rigid body
- Manipulator
- number of independently controllable joints

DOF of rigid body

${ }^{8}$ https://commons.wikimedia.org/wiki/File:6DOF.svg

## DOF examples



UR5 robot with Robotiq 3-finger gripper
6-DOF + 3-DOF gripper

$$
9
$$

## DOF examples (cont.)



DOF examples (cont.)


DOF examples (cont.)


PR2 service robot with Shadow C6 electrical hand 19-DOF $+20-$ DOF hand

DOF examples (cont.)


Boston Dynamics Atlas (2020)
28-DOF 10

[^1] hyperbole-it-is-sad-2c24a7f560ba
by input power source

- electrical
- hydraulic
- pneumatic
by field of work
- stationary
- arms with n DOF
- multi-finger hand
- mobile
- portal robot
- mobile platform
- running machines and flying robots
- anthropomorphic robots (humanoids)

Hopping robot

Salto Robot [2]


## Robot classification by mechanical structure

by mechanical structure


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[^2]- rotatory
- revolute
- translatory
- prismatic
- combinations
- spherical
- cylindrical
- planar
revolute joint

${ }^{12}$ https://www.youtube.com/playlist?list=PLrIVgT56nVQ4pm5QFeQ8Z288_VwopKmfW
prismatic joint


13
${ }^{13}$ https://www.youtube.com/playlist?list=PLrIVgT56nVQ4pm5QFeQ8Z288_VwopKmfW
joints with more than one degree of freedom


14
${ }^{14}$ https: //www.youtube.com/playlist?list=PLrIVgT56nVQ4pm5QFeQ8Z288_VwopKmfW

Robot classification by mechanical structure

by mechanical structure


by mechanical structure

- cartesian
- cylindrical
- spherical / polar
- Articulated Robot
- SCARA (Selective Compliance Assembly Robot Arm)

Robot classification by mechanical structure

Selective Compliance Assembly Robot Arm


## Task

Please find SCARA robots in the Fanuc industrial robot part.
${ }^{15}$ https: //www.youtube.com/watch?v=97KX-j8Onu0\&t=30s
by usage

- object manipulation
- object processing
- transport
- assembly
- quality testing
- deployment in non-accessible areas
- agriculture and forestry
- underwater
- building industry
- service robot in medicine, housework, ...



## Robotics is Fun!

- A dream of mankind:

Computers are the most ingenious product of human laziness to date.

$$
\text { computers } \Rightarrow \text { robots }
$$



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[^3]
## Outline

## Introduction

Spatial Description and Transformations
Rigid Body Configuration
Concatenation of rotation matrices
Homogenous Transformation
Transformation Equation
Forward Kinematics
Robot Description
Inverse Kinematics for Manipulators
Differential motion with homogeneous transformations
Jacobian
Trajectory planning
Trajectory generation
Dynamics

# Outline (cont.) 

Principles of Walking
Robot Control
Task-Level Programming and Trajectory Generation
Task-level Programming and Path Planning
Task-level Programming and Path Planning
Architectures of Sensor-based Intelligent Systems
Summary
Conclusion and Outlook

## Coordinate Systems

The pose of objects, in other words their position and orientation in Euclidian space can be described through specification of a cartesian coordinate system (B) in relation to a base coordinate system (A).


## Position:

- translation along the axes of the base coordinate system (A)

- given by position vector $\overrightarrow{\mathbf{A} \mathbf{P}}=\left[{ }^{A} p_{x},{ }^{A}{ }^{\prime} p_{y},{ }^{A} p_{z}\right]^{T} \in \mathcal{R}^{3}$

Orientation (in space):

- given by Rotation matrix $R_{B}=\left[\begin{array}{lll}\overrightarrow{X_{B}} & \overrightarrow{Y_{B}} & \overrightarrow{Z_{B}}\end{array}\right] \in \mathcal{R}^{3 \times 3}$
- given by Rotation matrix ${ }^{A} R_{B}=\left[{ }^{A} \vec{X}_{B}{ }^{A} \vec{Y}_{B}{ }^{A} \vec{Z}_{B}\right] \in \mathcal{R}^{3 \times 3}$
- ${ }^{A} R_{B}$ : the orientation of $B$ with respect to $A$. (Latex: \$^\{A\}R_\{B\}\$)
- ${ }^{A} \vec{X}_{B},{ }^{A} \vec{Y}_{B},{ }^{A} \vec{Z}_{B}$ are projection of $\overrightarrow{X_{B}}, \overrightarrow{Y_{B}}, \overrightarrow{Z_{B}}$ in A .


## Dot product

In terms of the geometric definition, the dot product of two unit vectors $\vec{a}$ and $\vec{b}$ means the projection of the $\vec{a}$ in $\vec{b}$.
$\vec{a} \cdot \vec{b}=\|a\|\|b\| \cos (\theta)$

$$
\begin{gathered}
A \vec{X}_{B}=\left[\begin{array}{l}
\overrightarrow{X_{B}} \cdot \overrightarrow{X_{A}} \\
\overrightarrow{X_{B}} \cdot \overrightarrow{Y_{A}} \\
\overrightarrow{X_{B}} \cdot \overrightarrow{Z_{A}}
\end{array}\right] \text { and }{ }^{A} R_{B}=\left[\begin{array}{lll}
A \vec{X}_{B} & A \vec{Y}_{B} & A \vec{Z}_{B}
\end{array}\right] \\
{ }^{A} R_{B}=\left[\begin{array}{lll}
\overrightarrow{X_{B}} \cdot \overrightarrow{X_{A}} & \overrightarrow{Y_{B}} \cdot \overrightarrow{X_{A}} & \overrightarrow{Z_{B}} \cdot \overrightarrow{X_{A}} \\
\overrightarrow{X_{B}} \cdot \overrightarrow{Y_{A}} & \overrightarrow{Y_{B}} \cdot \overrightarrow{Y_{A}} & \overrightarrow{Z_{B}} \cdot \overrightarrow{Y_{A}} \\
\overrightarrow{X_{B}} \cdot \overrightarrow{Z_{A}} & \overrightarrow{Y_{B}} \cdot \overrightarrow{Z_{A}} & \overrightarrow{Z_{B}} \cdot \overrightarrow{Z_{A}}
\end{array}\right]
\end{gathered}
$$

$$
\begin{aligned}
& { }^{A} R_{B}=\left[\begin{array}{lll}
\overrightarrow{X_{B}} \cdot \overrightarrow{X_{A}} & \overrightarrow{Y_{B}} \cdot \overrightarrow{X_{A}} & \overrightarrow{Z_{B}} \cdot \overrightarrow{X_{A}} \\
\overrightarrow{X_{B}} \cdot \overrightarrow{Y_{A}} & \overrightarrow{Y_{B}} \cdot \overrightarrow{Y_{A}} & \overrightarrow{Z_{B}} \cdot \overrightarrow{Y_{A}} \\
\overrightarrow{X_{B}} \cdot \overrightarrow{Z_{A}} & \overrightarrow{Y_{B}} \cdot \overrightarrow{Z_{A}} & \overrightarrow{Z_{B}} \cdot \overrightarrow{Z_{A}}
\end{array}\right] \text { the projection of } \overrightarrow{X_{A}} \text { in } B \\
& { }^{A} R_{B}=\left[\begin{array}{lll}
A_{X} \vec{X}_{B} & { }^{A} \vec{Y}_{B} & A \vec{Z}_{B}
\end{array}\right]=\left[\begin{array}{c}
B \vec{X}_{A}^{T} \\
{ }^{B} \vec{Y}_{A}^{T} \\
{ }_{B} \vec{Z}_{A}^{T}
\end{array}\right]=\left[\begin{array}{lll}
B \vec{X}_{A} & B \vec{Y}_{A} & B \vec{Z}_{A}
\end{array}\right]^{T}={ }^{B} R_{A}^{T}
\end{aligned}
$$

$$
{ }^{A} R_{B}=\left[\begin{array}{lll}
A^{{ }_{X}} & A \vec{Y}_{B} & A \vec{Z}_{B}
\end{array}\right]=\left[\begin{array}{c}
B \vec{X}_{A}^{T} \\
B \vec{Y}_{A}^{T} \\
B \vec{Z}_{A}^{T}
\end{array}\right]=\left[\begin{array}{lll}
B \vec{X}_{A} & B \vec{Y}_{A} & B \vec{Z}_{A}
\end{array}\right]^{T}={ }^{B} R_{A}^{T}
$$

The inverse of a rotation matrix is simply its transpose:

$$
{ }^{A} R_{B}^{-1}={ }^{B} R_{A}={ }^{B} R_{A}^{T} \quad \text { and } \quad{ }^{A} R_{B}^{B} R_{A}=1
$$

whereas $I$ is the identity matrix.

- Position:
- given through $\overrightarrow{A P} \in \mathcal{R}^{3}$
- Orientation:
- given through the projection of $\overrightarrow{X_{B}}, \overrightarrow{Y_{B}}, \overrightarrow{Z_{B}} \in \mathcal{R}^{3}$ of $B$ to the origin system $A$
- summarized to rotation matrix ${ }^{A} R_{B}=\left[{ }^{A} \vec{X}_{B}{ }^{A} \vec{Y}_{B}{ }^{A} \vec{Z}_{B}\right] \in \mathcal{R}^{3 \times 3}$

$$
{ }^{A} R_{B}=\left[\begin{array}{lll}
r_{11} & r_{21} & r_{31} \\
r_{12} & r_{22} & r_{32} \\
r_{13} & r_{23} & r_{33}
\end{array}\right]
$$

- redundant, since there are 9 parameters for 3 degrees of freedom

Write the Rotation matrix of ${ }^{A} R_{B}$.
${ }^{A} R_{B}=\left[{ }^{A} \vec{X}_{B}{ }^{A} \vec{Y}_{B}{ }^{A} \vec{Z}_{B}\right]$
${ }^{A} R_{B}=\left[\begin{array}{ccc}1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0\end{array}\right]$


Sequential multiplication of the rotation matrices by order of rotation.

1. rotation $\varphi$ (phi) around the $x$-axis $R_{x, \varphi}-$ Roll
2. rotation $\theta$ (theta) around the $y$-axis $R_{y, \theta}$ - Pitch
3. rotation $\psi$ (psi) around the $z$-axis $R_{z, \psi}-$ Yaw

(shortened representation: $S: \sin , C: \cos$ )
The rotation matrix corresponding to a rotation around the $x$-axis with angle $\varphi$ (phi):

$$
R_{x, \varphi}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & C \varphi & -S \varphi \\
0 & S \varphi & C \varphi
\end{array}\right]
$$

The rotation matrix corresponding to a rotation around the $y$-axis with angle $\theta$ (theta):

$$
R_{y, \theta}=\left[\begin{array}{ccc}
C \theta & 0 & S \theta \\
0 & 1 & 0 \\
-S \theta & 0 & C \theta
\end{array}\right]
$$

The rotation matrix corresponding to a rotation around the $z$-axis with angle $\psi(p s i)$ :

$$
R_{z, \psi}=\left[\begin{array}{ccc}
C \psi & -S \psi & 0 \\
S \psi & C \psi & 0 \\
0 & 0 & 1
\end{array}\right]
$$

$$
\begin{gathered}
R_{\psi, \theta, \varphi}=R_{z, \psi} R_{y, \theta} R_{x, \varphi} \\
=\left[\begin{array}{ccc}
C \psi & -S \psi & 0 \\
S \psi & C \psi & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{ccc}
C \theta & 0 & S \theta \\
0 & 1 & 0 \\
-S \theta & 0 & C \theta
\end{array}\right]\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & C \varphi & -S \varphi \\
0 & S \varphi & C \varphi
\end{array}\right] \\
=\left[\begin{array}{ccc}
C \psi C \theta & C \psi S \theta S \varphi-S \psi C \varphi & C \psi S \theta C \varphi+S \psi S \varphi \\
S \psi C \theta & S \psi S \theta S \varphi+C \psi C \varphi & S \psi S \theta C \varphi-C \psi S \varphi \\
-S \theta & C \theta S \varphi & C \theta C \varphi
\end{array}\right]
\end{gathered}
$$

Remark: Matrix multiplication is not commutative:

$$
A B \neq B A
$$

- Several rotations can be multiplied. The following applies:
- If the rotations are performed in relation to the current, newly defined (or changed) coordinate system, the newly added transformation matrices need to be multiplicatively appended on the right-hand side.
- If all of them are performed in relation to the fixed reference coordinate system, the transformation matrices need to be multiplicatively appended on the left-hand side.

Mapping: changing descriptions from frame to frame.
For example, change the reference frame of $B \vec{P}_{1}$ ?

$$
\begin{aligned}
A_{P_{1}} & =\left[\begin{array}{l}
B \vec{X}_{A} \cdot{ }^{B} \vec{P}_{1} \\
B \vec{Y}_{A} \cdot \vec{P}_{1} \\
B \vec{Z}_{A} \cdot \vec{P}_{1}
\end{array}\right] \\
& =\left[\begin{array}{l}
B \vec{X}_{A}^{T} \\
B \vec{Y}_{A}^{T} \\
B \vec{Z}_{A}^{T}
\end{array}\right] \cdot{ }^{B} \vec{P}_{1} \\
& ={ }^{A} R_{B} \vec{P}_{1}
\end{aligned}
$$



Three common uses of a rotation matrix:

- represent an orientation
- rotate a vector or frame
- change the frame of reference of a vector or frame
- Homogeneous transformation matrix:

$$
T=\left[\begin{array}{ll}
R & \vec{p} \\
P & S
\end{array}\right]
$$

where $P$ depicts the perspective transformation and $S$ the scaling.

- In robotics, $P=\left[\begin{array}{lll}0 & 0 & 0\end{array}\right]$ and $S=1$. Other values are used for computer graphics.
- Combination of $\vec{p}$ and $R$ to $T=\left[\begin{array}{cc}R & \vec{p} \\ \overrightarrow{0} & 1\end{array}\right] \in \mathcal{R}^{4 \times 4}$
- Concatenation of several $T$ through matrix multiplication
- ${ }^{A} T_{B}{ }^{B} T_{C}={ }^{A} T_{C}$
- not commutative, in other words ${ }^{B} T_{C}{ }^{A} T_{B} \neq{ }^{A} T_{B}{ }^{B} T_{C}$

They are represented as four vectors using the elements of homogeneous transformation.

$$
T=\left[\begin{array}{cccc}
\mathbf{r}_{1} & \mathbf{r}_{2} & \mathbf{r}_{3} & \mathbf{p}  \tag{1}\\
0 & 0 & 0 & 1
\end{array}\right]=\left[\begin{array}{cccc}
r_{11} & r_{21} & r_{31} & p_{x} \\
r_{12} & r_{22} & r_{32} & p_{y} \\
r_{13} & r_{23} & r_{33} & p_{z} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The inverse of a rotation matrix is simply its transpose:

$$
R^{-1}=R^{T} \text { and } R R^{T}=I
$$

whereas $/$ is the identity matrix.
The inverse of (1) is:

$$
T^{-1}=\left[\begin{array}{cccc}
r_{11} & r_{12} & r_{13} & -\mathbf{p}^{\boldsymbol{\top}} \cdot \mathbf{r}_{1} \\
r_{21} & r_{22} & r_{23} & -\mathbf{p}^{\boldsymbol{\top}} \cdot \mathbf{r}_{2} \\
r_{31} & r_{32} & r_{33} & -\mathbf{p}^{\boldsymbol{\top}} \cdot \mathbf{r}_{3} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

whereas $\mathbf{r}_{1}, \mathbf{r}_{2}, \mathbf{r}_{3}$ and $\mathbf{p}$ are the four column vectors of (1) and $\cdot$ represents the dot product of vectors.

A translation with a vector $\left[p_{x}, p_{y}, p_{z}\right]^{T}$ is expressed through a transformation:

$$
T_{\left(p_{x}, p_{y}, p_{z}\right)}=\left[\begin{array}{cccc}
1 & 0 & 0 & p_{x} \\
0 & 1 & 0 & p_{y} \\
0 & 0 & 1 & p_{z} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The transformation corresponding to a rotation around the $x$-axis with angle $\varphi$ (phi):

$$
T_{x, \varphi}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & C \varphi & -S \varphi & 0 \\
0 & S \varphi & C \varphi & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The transformation corresponding to a rotation around the $y$-axis with angle $\theta$ (theta):

$$
T_{y, \theta}=\left[\begin{array}{cccc}
C \theta & 0 & S \theta & 0 \\
0 & 1 & 0 & 0 \\
-S \theta & 0 & C \theta & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The transformation corresponding to a rotation around the $z$-axis with angle $\psi$ ( $p s i$ ):

$$
T_{z, \psi}=\left[\begin{array}{cccc}
C \psi & -S \psi & 0 & 0 \\
S \psi & C \psi & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

- Transform of Coordinate systems: frame: a reference $S$ typical frames:
- robot base
- end effector
- table (world)
- 
- object
- camera

One has the following transformations:

- Z:

World $\rightarrow$ Manipulator base

- $T_{6}$ :

Manipulator base $\rightarrow$ Manipulator end

- E:

Manipulator end $\rightarrow$ End effector

- B:

World $\rightarrow$ Object

- G:

Object $\rightarrow$ End effector

There are two descriptions for the desired end effector position, one in relation to the object and the other in relation to the manipulator. Both descriptions should equal to each other for grasping:


In order to find the manipulator transformation:

$$
T_{6}=Z^{-1} B G E^{-1}
$$

In order to determine the position of the object:

$$
B=Z T_{6} E G^{-1}
$$

This is also called kinematic chain.

## Example: coordinate transformation



## Example: coordinate transformation

Given $T_{\text {Base-Apriltag }}, T_{\text {Camera-Apritag }}, T_{\text {Camera-Object }}$, calculate $T_{\text {Base-Object }}$.


$$
T_{\text {Base-Object }}=T_{\text {Base-Apriltag }} T_{\text {Camera-Apritag }}^{-1} T_{\text {Camera-Object }}
$$

- A homogeneous transformation depicts the position and orientation of a coordinate frame in space.
- If the coordinate frame is defined in relation to a solid object, the position and orientation of the solid object is unambiguously specified.
- Three common uses of a transformation matrix: to represent a rigid-body configuration; to change the frame of reference of a vector or a frame; to displace a vector or a frame.
- Several translations and rotations can be multiplied.
- right-hand multiplication $\rightarrow$ in relation to thecurrent, newly defined (or changed) coordinate system, .
- left-hand multiplication $\rightarrow$ in relation to the fixed reference coordinate system.
- Joint coordinates:

A vector $\mathbf{q}(t)=\left(q_{1}(t), q_{2}(t), \ldots, q_{n}(t)\right)^{T}$ (a robot configuration)

- End effector coordinates (Object coordinates):
- A vector $\mathbf{p}=\left[p_{x}, p_{y}, p_{z}\right]^{T}$
- Rotation matrix:

$$
R=\left[\begin{array}{lll}
r_{11} & r_{12} & r_{13} \\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{array}\right]
$$

Outlook
Spatial Description and Transformations - Transformation Equation


- Can we use less of 9 parameters to represent the orientation?
- How to construct the transformation matrix of the manipulator's end-effector relative to the base of the manipulator?
- Read (available on google \& library):
- J. F. Engelberger, Robotics in service. MIT Press, 1989
- K. Fu, R. González, and C. Lee, Robotics: Control, Sensing, Vision, and Intelligence. McGraw-Hill series in CAD/CAM robotics and computer vision, McGraw-Hill, 1987
- R. Paul, Robot Manipulators: Mathematics, Programming, and Control: the Computer Control of Robot Manipulators. Artificial Intelligence Series, MIT Press, 1981
- J. Craig, Introduction to Robotics: Pearson New International Edition: Mechanics and Control. Always learning, Pearson Education, Limited, 2013
- Repeat your linear algebra knowledge, especially regarding elementary algebra of matrices.


## Bibliography

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[3] J. F. Engelberger, Robotics in service. MIT Press, 1989.
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