## Introduction to Robotics

## Assignment \#2

Due: 07.05.2019, 23:59

Task 2.1 (4 points) Planar manipulator: Consider the planar robot manipulator shown in figure 1 with the joint angles $\theta_{1}, \theta_{2}$ and $\theta_{3}$ constrained by the following relation: $\theta_{3}=180^{\circ}-\theta_{1}-\theta_{2}$.


Figure 1: 3-joint planar manipulator.
2.1.1 (2 points): Determine the partial homogeneous transformations ${ }^{i-1} A_{i}, i=1,2,3$ for each of the coordinate frames shown in figure 1 and show the planar manipulator transformation ${ }^{0} T_{3}=$ ${ }^{0} A_{1}{ }^{1} A_{2}{ }^{2} A_{3}$ to be equal to

$$
{ }^{0} T_{3}=\left[\begin{array}{cccc}
-1 & 0 & 0 & C_{1} a_{1}-C_{3} a_{2}-a_{3} \\
0 & -1 & 0 & S_{1} a_{1}+S_{3} a_{2} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

with $C_{i} \equiv \cos \left(\theta_{i}\right)$ and $S_{i} \equiv \sin \left(\theta_{i}\right)$. Write down intermediate steps and interpret the solution.
Hint: Use the following trigonometric identities to simplify the resulting transformation matrices:

$$
\begin{aligned}
\cos \left(\theta_{1}+\theta_{2}\right) & =\cos \left(\theta_{1}\right) \cos \left(\theta_{2}\right)-\sin \left(\theta_{1}\right) \sin \left(\theta_{2}\right) \\
\sin \left(\theta_{1}+\theta_{2}\right) & =\sin \left(\theta_{1}\right) \cos \left(\theta_{2}\right)+\cos \left(\theta_{1}\right) \sin \left(\theta_{2}\right)
\end{aligned}
$$

2.1.2 (2 points): Specify the two additional homogeneous transformations that are required in order to facilitate a rotation of the manipulator around the axes $x_{0}$ (angle $\theta_{0}$ ) and $x_{3}$ (angle $\theta_{4}$ ). It is sufficient to explicitly specify both homogeneous transformations without recalculation of the full manipulator transformation.

Task 2.2 (3 points) DH-Parameter parallel joints: Figure 2(a) shows a 3-joint planar manipulator. Figure 2(b) shows the rotation axes of the three joints to be parallel to each other. Visually specify the coordinate frame of each joint and determine the corresponding DH parameters.


Figure 2: 3-joint planar manipulator.

Task 2.3 (3 points) DH-Parameter Stanford manipulator: The Stanford manipulator has five revolute joints and one prismatic joint. Figure 3(a) illustrates the general idea and figure 3(b) shows the general configuration of a Stanford manipulator. Assume the distance between joint 1 and joint 2 to be $d_{2}$. Specify the coordinate frame of each joint in the drawing and present the corresponding Denavit-Hartenberg ( DH ) parameters as a table.


Figure 3: Stanford manipulator.

Task 2.4 (3 points) DH-Parameter from URDF: Extract the DH-parameter for the 4-DOF nonplanar manipulator which will be used in the Robot Practical Course. The URDF file can be found on the TAMS website:
http://tams.informatik.uni-hamburg.de/lectures/2019ss/vorlesung/itr/doc/
4dofnonplanar.urdf


Figure 4: 4-DOF non-planar manipulator.
You can also use the tool from the practical course to visualize. Update your git repository (run git pull in $\sim /$ catkin_ws/src/itr_rpc, then run the visualization with roslaunch itr_rpc assignment_2.4.launch)

Task 2.5 (7 points) DH-Parameter SCARA: An important type of manipulator is the SCARA type, a manipulator with four vertically aligned joint axes. Figure 5 shows the joint coordinate frames of such a manipulator (Adept One).


Figure 5: SCARA manipulator (Adept One).
The joint variable vector is defined as $\mathbf{q}=\left[\theta_{1}, \theta_{2}, d_{3}, \theta_{4}\right]^{T}$. The kinematic parameters can be found in table 1:

| Link | $\theta$ | $d$ | $a$ | $\alpha$ | Zero position |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $q_{1}$ | $d_{1}$ | $a_{1}$ | $\pi$ | 0 |
| 2 | $q_{2}$ | 0 | $a_{2}$ | 0 | 0 |
| 3 | 0 | $d_{3}$ | 0 | 0 | 100 |
| 4 | $q_{4}$ | $d_{4}$ | 0 | 0 | $\pi / 2$ |

Table 1: Kinematic parameters of the SCARA manipulator.
The "Adept One" SCARA manipulator has following values for $d$ and $a$ :

$$
\begin{aligned}
& d=\left[877,0, d_{3}, 200\right]^{T} \quad \mathrm{~mm} \\
& a=[425,375,0,0]^{T} \quad \mathrm{~mm}
\end{aligned}
$$

2.5.1 (2 points): Verify the manipulator shown in Figure 5 according to the Denavit-Hartenberg convention. (For each rule and requirement of the DH convention, explain where it has been used and why.)
2.5.2 (3 points): Determine the homogeneous transformation ${ }^{\text {Base }} T_{\text {Tool }}$ of the given manipulator.
2.5.3 ( 2 points): Determine the location of the tool center point, given the following joint value vector:

$$
q=[\pi / 4,-\pi / 3,120, \pi / 2]^{T}
$$

