



64-424 Intelligent Robotics

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Technical Aspects of Multimodal Systems

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Outline

1. Fuzzy logic



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Introduction

The concept of **fuzzy logic** was introduced by Lotfi Zadeh (1965)

- ▶ Inspired by human information processing capabilities
- ▶ People do not require precise, numerical information input, yet they are capable of highly adaptive control

General assumption

- ▶ If feedback controllers could be programmed to accept noisy, imprecise input, they would be much more effective and perhaps easier to implement
- ▶ How important is it to be exactly right when a rough answer will do?



Fuzzy control (cont.)

- ▶ *Fuzzy* means: blurred, diffuse, vague, uncertain, ...
- ▶ Fuzzy control uses fuzzy sets as mechanism for
 - ▶ Abstraction of unnecessary or too complex details
 - ▶ Troubleshooting of problems which are not easily solvable by a simple *yes* or *no* decision
 - ▶ Modeling of (*soft*) concepts without any sharp borders



Fuzzy control (cont.)

- ▶ Unsharp linguistic grading of terms like "**big**", "**beautiful**", "**strong**" ...
- ▶ Human thinking models and behavior models with first-level logic
 - ▶ **Car driving:** *if-then-rules*
 - ▶ **Car parking:** Accurate up to a millimeter?
- ▶ Fuzzy speech instead of numerical description
 - ▶ "Brake 2.52 m ahead of the curve!" → only in machine systems
 - ▶ "Brake shortly before the curve!" → in natural language



Adaptive control methods

Fuzzy control is generally a good fit for realization of *adaptive control methods*

- ▶ Control can be understood as mapping from a sensor space onto actions
- ▶ In many cases it is *a priori* unknown, which measurement parameters are especially important for the choice of actions
- ▶ Some systems are very hard to describe in a mathematical way
- ▶ Often, sensor data is inaccurate, noisy and/or high dimensional



Adaptive control methods (cont.)

Models for adaptive control systems

- ▶ The creation of an ideal mapping between sensor space and actions is very difficult with classical methods of control engineering
- ▶ In order to control such systems, a simpler method needs to be used for description
- ▶ Neural networks
- ▶ Fuzzy-Controller



Linguistic variables

Linguistic variables are one of the main building blocks of fuzzy logic

- ▶ A *linguistic variable* is a variable, which can take on a range of **linguistic terms**
- ▶ A *linguistic term* (*value, label*) is the quantification of a term from natural language through a fuzzy set
- ▶ Many terms of natural language can be characterized through degree of membership related to fuzzy sets
- ▶ Therefore, fuzzy sets can be considered as the basic tool for modelling of *linguistic terms*



Linguistic variables (cont.)

Examples:

- ▶ Linguistic variable: **"SPEED"**
- ▶ Linguistic terms of **"SPEED"**
"high", "low", "rapid", "economical"
- ▶ Linguistic variable: **"BUILDING"**
- ▶ Linguistic terms of **"BUILDING"**
"cottage", "bungalow", "skyscraper"



Linguistic variables (cont.)

A **linguistic variable** is characterized through a quintuplet

$$(v, T(v), X, G, M)$$

With:

v : the name of the variable

$T(v)$: a set of linguistic terms of v , whereas each value is a fuzzy set in the universe X

G : a syntax rule, which creates $T(v)$ from a set of basic terms

M : a semantic rule, which maps its meaning to each value of $T(v)$



Characteristic function

Crisp sets can be defined through specification of their characteristic function:

$$\mu_{\mathcal{A}}(x) = \begin{cases} 1 & \text{for } x \in \mathcal{A} \\ 0 & \text{for } x \notin \mathcal{A}, \end{cases}$$

where $\mu_{\mathcal{A}} : X \rightarrow \{0, 1\}$



Membership function

For **fuzzy sets** A , a generalized characteristic function μ_A is used, which maps a real number $[0, 1]$ to each element $x \in X$:

$$\mu_A : X \rightarrow [0, 1]$$

- ▶ The function μ_A is called **membership function** (MF)
- ▶ It indicates the "degree", to which the element x belongs to the described unsharp set A (\rightarrow fuzzy set)



Membership function (cont.)

Representation of membership functions

- ▶ Discrete representation
 - ▶ Fixed-size array
 - ▶ Saving of the MF-Values for the whole x -codomain

- ▶ Parametric representation
 - ▶ Functions with parameters (less space required)
 - ▶ Typical types: Singleton, triangular shape, trapezoid shape, bell curve, B-Spline basis function



Membership function (cont.)

Creation of the membership functions

- ▶ Context-dependent specification
 - ▶ Experimental, domain- and application specific
- ▶ Construction using sample data
 - ▶ Clustering
 - ▶ Interpolation
 - ▶ Curve Fitting (Least-squares)
 - ▶ Neural networks
- ▶ Knowledge acquisition through experts
 - ▶ One or several experts
 - ▶ Directly and indirectly



Fuzzy set

A **fuzzy set** A over a universe X is given through a mapping $\mu_A : X \rightarrow [0, 1]$.

For all $x \in X$, $\mu_A(x)$ denotes the degree of affinity (membership) of x in A

The set of all elements from X with positive affinity to A is called **support** of a fuzzy set:

$$\text{support}(\mu_A(x)) = \{x \in X \mid \mu_A(x) > 0\}$$



Fuzzy set (cont.)

Example:

- ▶ The set of integer numbers approximately equal to 10:

$$A_{10} = (0.1, 7), (0.5, 8), (0.8, 9), (1.0, 10), (0.8, 11), (0.5, 12), (0.1, 13)$$



Fuzzy control

Main principles of fuzzy control

- ▶ Description of the desired control behaviour by means of colloquial, qualitative rules
- ▶ Quantification of linguistic values through fuzzy sets
- ▶ Rule evaluation through fuzzy logic methods (*fuzzy inference*) or interpolation



Fuzzy control (cont.)

In a fuzzy control system, the influence on dynamic circumstances of a fuzzy system is characterized by a set of linguistic description rules

IF (a set of conditions is satisfied)

THEN (a set of consequences can be inferred)

Conditions (antecedents or premises) of the IF-part:

→ Linguistic variables from the domain of process states

Conclusions (consequences) of the THEN-part:

→ Linguistic variables from the control domain



Fuzzy control (cont.)

Main goals of fuzzy control

- ▶ **Intelligent control**
 - ▶ Use of expert knowledge
- ▶ **Linguistic control**
 - ▶ Control specifics are transparent
 - ▶ Advantageous for the human-machine interface
- ▶ **Parallel control**
 - ▶ Modularization
 - ▶ High processing performance



Fuzzy control (cont.)

Advantages

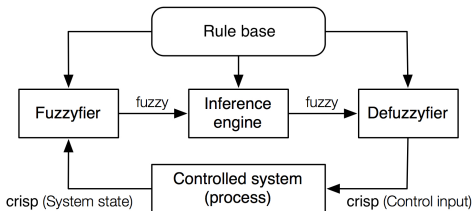
- ▶ Controller design is possible without any special knowledge about models of specific domains
- ▶ Controller design is efficient (up to a certain degree)
- ▶ Complies with real-time requirements
- ▶ Robust, even when using cheap and noisy sensors



Components of fuzzy control

A complete fuzzy controller consists of four major components

- ▶ Rule base
- ▶ Fuzzifier
- ▶ Inference engine
- ▶ Defuzzifier





Rule base

The rule base is the central component that stores expert knowledge

- ▶ It contains the **control strategies** in the form of IF-THEN rules

The rule base is considered a part of the knowledge base of the fuzzy controller, other components being:

- ▶ The **input membership functions**
 → required for *fuzzification* of crisp input values
- ▶ The **output membership functions**
 → required for *defuzzification* of the inference result(s)



Rule base (cont.)

Example:

Given a fuzzy control system with two inputs A and B and a single output C , the general representation of the rule base would be:

R_1 : IF (x is A_1 OR y is B_1) THEN (z is C_1)

R_2 : IF (x is A_2 OR y is B_2) THEN (z is C_2)

...

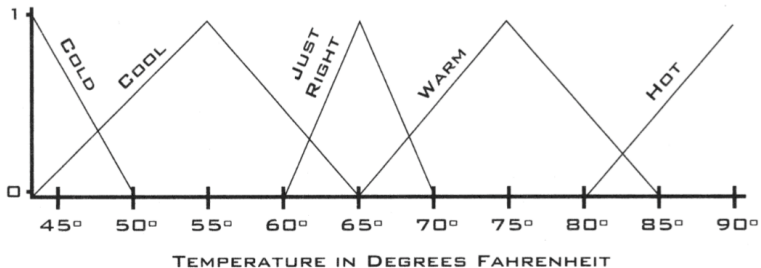
R_k : IF (x is A_k OR y is B_k) THEN (z is C_k)

Note: Inputs and outputs mentioned here are strictly fuzzy.



Fuzzifier

The fuzzifier converts the crisp input values into fuzzy representations based on the membership degree to established fuzzy sets





Fuzzifier (cont.)

- ▶ The established fuzzy sets and associated membership functions are designed to exploit the inherent inaccuracy of input data (e.g. from sensors)
- ▶ The fuzzifier approximates the human reasoning process
- ▶ While expert knowledge of the process to be controlled is helpful, it is not a requirement
- ▶ The overall implementation effort of the controller is reduced significantly



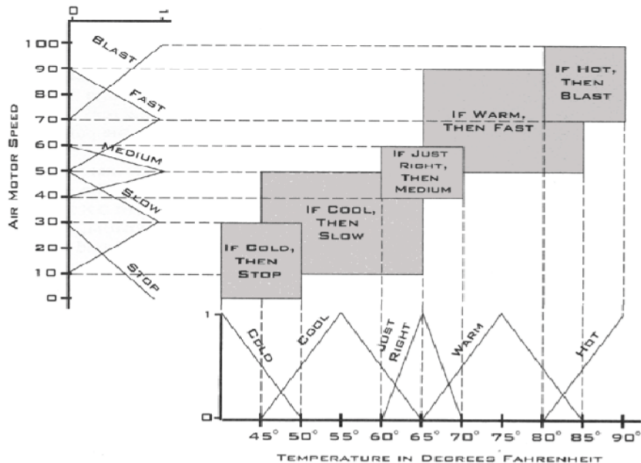
Inference engine

The inference engine processes the *fuzzified* input values based on evaluation of the rule base

- ▶ All rules within the rule base are evaluated
- ▶ The evaluation usually occurs in parallel (depends on implementation)
- ▶ All evaluated rules contribute to the fuzzy output value to some degree
- ▶ The contribution of most rules to the output value is 0
- ▶ The resulting fuzzy output value is a *union* of the results of all evaluated rules



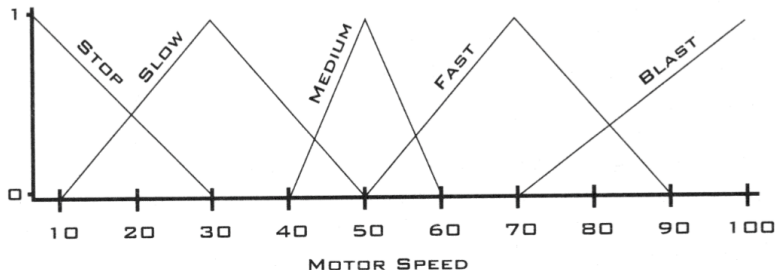
Inference engine (cont.)





Defuzzifier

The defuzzifier converts the obtained fuzzy output value into a crisp representation based on the output membership functions





Defuzzifier (cont.)

Several strategies exist for defuzzification

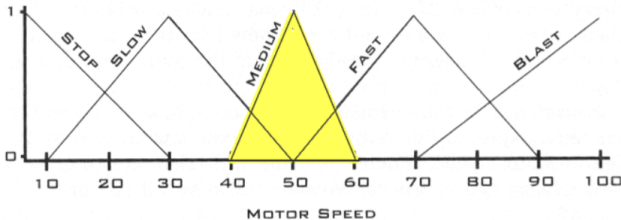
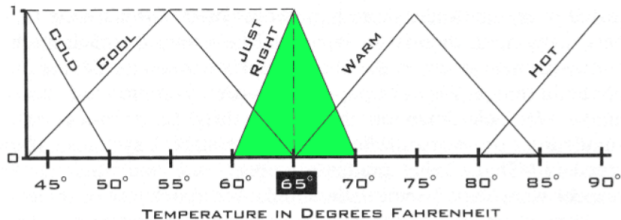
- ▶ The **center of gravity (CoG)** technique is very common

Other strategies include:

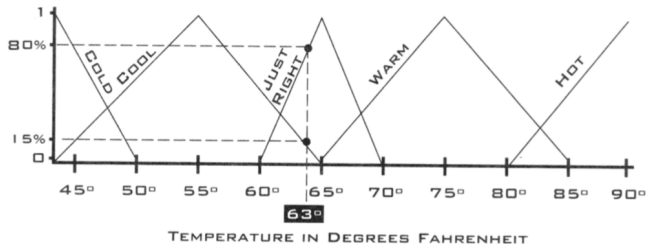
- ▶ **Mean of maximum**
 - The defuzzified result represents the mean value of all actions, whose membership functions reach the maximum
- ▶ **Weighted average method**
 - Formed by weighting each output by its respective maximum membership degree
- ▶ ...



Simple case

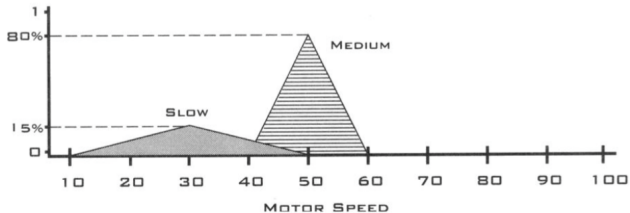
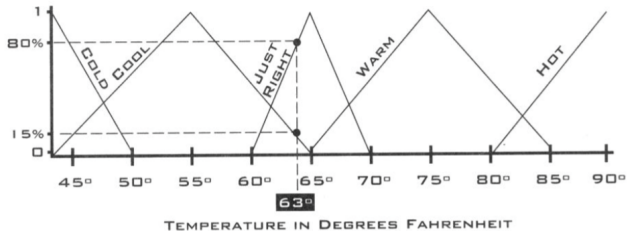


General case



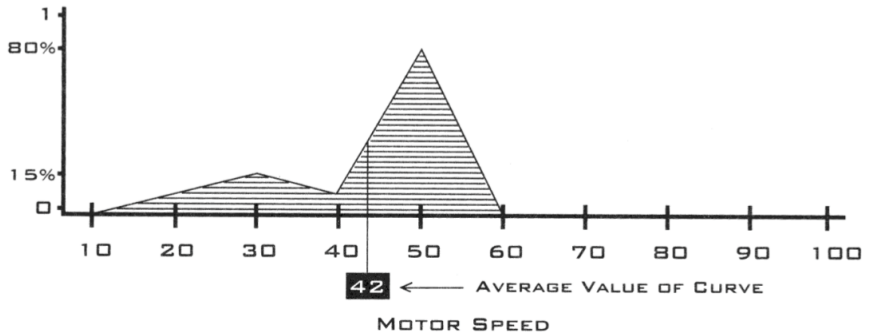
- ▶ IF temperature is **cool** THEN motor speed is **slow**
- ▶ IF temperature is **just right** THEN motor speed is **medium**

General case (cont.)





General case (cont.)





Curse of dimensionality

Fuzzy logic based control models (nonlinear modeling techniques) are affected by the *curse of dimensionality*

→ If the number of inputs grows, the cost of both implementing the rule base and obtaining an output value increase **exponentially**.



Summary

- ▶ Fuzzy logic provides a different way to approach a control problem
- ▶ It is based on natural language
- ▶ It allows to solve the problem without the need of a mathematical model
- ▶ FL based controllers require less implementation effort and are thus usually cheaper
- ▶ FL is inherently tolerant of imprecise data
- ▶ It can model nonlinear functions of arbitrary complexity
- ▶ Fuzzy logic can be combined with conventional control techniques



Summary (cont.)

- ▶ Fuzzy logic is not a cure-all!
- ▶ Fuzzy logic is a convenient way to map an input space to an output space
- ▶ However, many controllers can do a fine job without it
- ▶ Fuzzy logic *can* be a powerful tool for dealing with imprecision and nonlinearity



Literature list

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