

## Introduction to Robotics

### Assignment #5

Due: 19.06.2018, 23:59

Preferred tools/languages for programming, plotting and solving are

python, matlab, octave or gnuplot.

To use Matlab on the pool computers at group TAMS, type

```
source /informatik2/tams/software/profile.d/matlab.sh
```

and run `matlab` from console.

Always provide source code as well as all intermediate steps. Make sure to annotate your code in detail in order to provide readability and fast understanding. "Here, some magic happens" will cost you points.

**Task 5.1 (4 points) Basis-Splines – Direct Computation:** In the lecture the recursive calculation of the basis splines has been introduced. With regard to real-time time critical applications, direct computation for a given order has advantages. Deduce the explicit formula for the basis splines with the degree 1, 2 and 3.

**Task 5.2 (2 points) Basis-Splines – Uniform Visualization:** Draw the basis splines of the order 1 to 4 within the intervals  $[t_i, t_{i+1}]$ ,  $i = 0, 1, 2, 3, 4$ . Provide the function for the first spline for each order within your solution as well as the attached source code.

**Task 5.3 (4 points) Basis-Splines – Non-uniform Visualization:** Draw the resulting curve for a B-Spline of 4<sup>th</sup> order for the knot vector  $[0 \ .3 \ .4 \ .8 \ 1]$  (time intervals). Provide functions and plot within your solution as well as the attached source code.

**Task 5.4 (4 points) Lagrange polynomial:** Given a set of data points  $(0, 8), (2, -4), (5, 2), (9, 0)$ . Calculate the Lagrange polynomial  $P_3(x)$  through the data points.

Simplify the equation as much as possible. Use a tool of your choice (gnuplot is preferred) to visualize the four Lagrange basis polynomials and the polynomial  $p_3(x)$ .

**Task 5.5 (6 points) PID-controller:** Consider a DC motor with

- J** moment of inertia of the rotor =  $0.1 \text{ kg m}^2$
- b** motor viscous friction constant =  $0.1 \text{ N m s}$
- K** motor torque constant =  $0.01 \text{ N m/Amp}$
- R** electric resistance =  $1 \text{ Ohm}$
- L** electric inductance =  $0.5 \text{ H}$

**5.5.1 (4 points):** Calculate the response of a PID-controller with the factors  $k_p$ ,  $k_i$  and  $k_d$  for the DC motor<sup>1</sup> with a 1-rad/sec step reference and plot the function for both subtasks with

1.  $k_p = 20, k_i = 0, k_d = 0,$
2.  $k_p = 500, k_i = 0, k_d = 0$  and
3.  $k_p = 100, k_i = 50, k_d = 0$

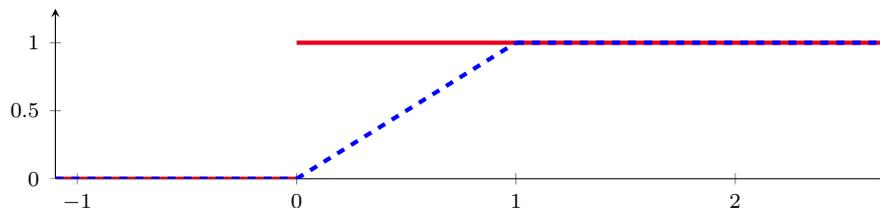
using a tool of your choice (matlab should be preferred).

to the step function:

$$y(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

the ramp function

$$y(t) = \begin{cases} 0, & t < 0 \\ t, & 0 \leq t < 1 \\ 1, & t \geq 1 \end{cases}$$



**5.5.2 (2 points):** Explain the impact of  $k_p$  and  $k_i$  and their combination. What will be improved if  $k_d \neq 0$ ?

**5.5.3 \*Bonus\* (4 points):** find  $k_p, k_i, k_d$  for the step function such that

- settling time is less than 2 seconds
- overshoot is less than 5%
- steady-state error less than 1%

<sup>1</sup>Find the transfer function of a DC motor to be  $\frac{K}{(J \cdot s + b) \cdot (L \cdot s + R) + K^2}$