

## Introduction to Robotics

### Assignment #1

Due: 17.04.2017, 23:59

**Task 1.1 (8 points) Pyramid:** A pyramid (square base  $AB = BC = CD = DA = 320\text{ mm}$ ; plumb-line  $ME = 615\text{ mm}$ , with vertex  $E$  located at the top and point  $M$  located at the center of the base) is held by a robot so that its square base  $ABCD$  is located in the  $xy$ -plane of a cartesian world coordinate frame  $M_{xyz}$ , with point  $M$  at its origin, the edges  $AB$  and  $CD$  parallel to the  $x$ -axis and the edges  $BC$  and  $AD$  parallel to the  $y$ -axis. Attached to the pyramid is an object coordinate frame  $M_{uvw}$ , which initially coincides with  $M_{xyz}$ . Write down the general formula for each rotation.

**1.1.1 (4 points):** Determine the locations of the vertices  $A$  through  $E$ , after the following sequence of rotations has been performed by the robot:

1. Rotation by  $\psi = -30^\circ$  around  $M_w$
2. Rotation by  $\phi = 115^\circ$  around  $M_u$
3. Rotation by  $\theta = 285^\circ$  around  $M_v$

**1.1.2 (4 points):** Same sequence of rotations, but using the rotation axes  $M_z$ ,  $M_x$  and  $M_y$  instead.

**Task 1.2 (6 points) Homogeneous transformations:** Given are three frames  $A$ ,  $B$  and  $C$  as well as the following two homogeneous transformations:

$${}^A T_B = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$${}^B T_C = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 2 \\ 1/2 & \sqrt{3}/2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**1.2.1 (3 points):** Can the interpretation of the transformation  ${}^A T_C$  be considered to be unambiguous? Explain your answer.

**1.2.2 (3 points):** Visualize the three coordinate systems with a tool of your choice.

**Task 1.3 (6 points) Euler angles:**

**1.3.1 (4 points):** Give four examples of Euler angle combinations  $(\phi, \theta, \psi)$  and interpret their geometric meaning using natural language.

"This is a rotation around  $x$  by  $\phi$ " is not sufficient. Explain the properties of the transformation.

**1.3.2 (2 points):** There are 12 possible sequences of rotations with Euler-angles around the axes (see slide 29). Explain why there are exactly 12!