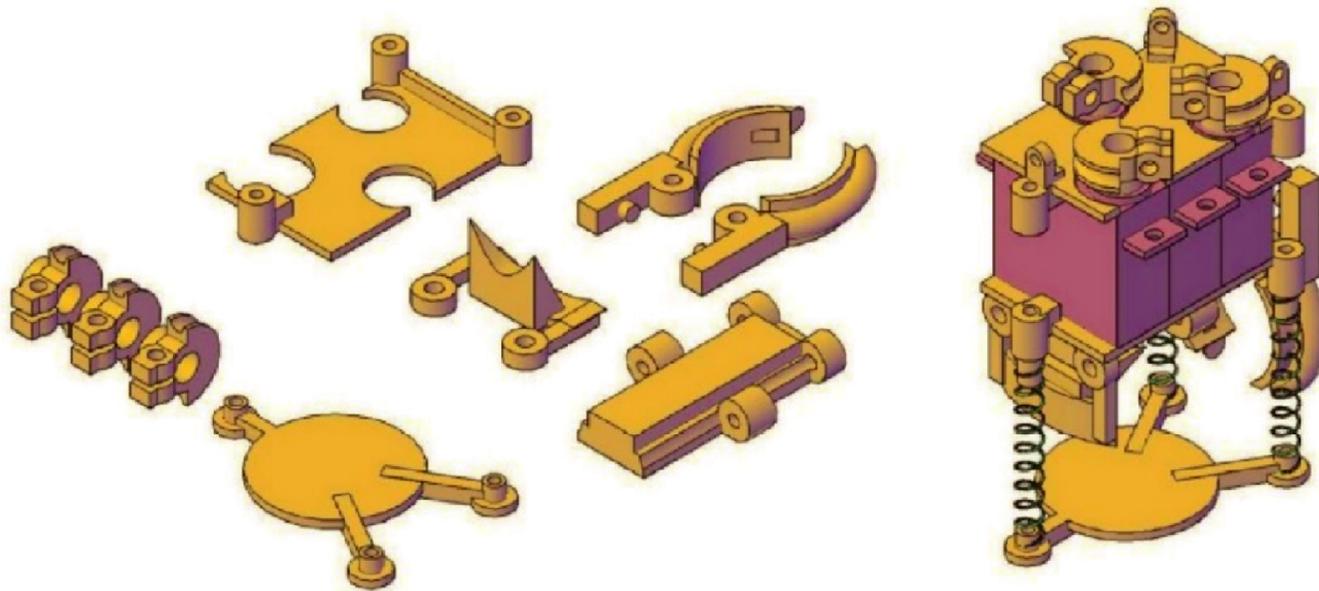


Dynamics Analysis for a 3-PRS Spatial Parallel Manipulator-Wearable Haptic Thimble

Masoud Moeini, University of Hamburg, Oct 2016



[Wearable Haptic Thimble,A Developing Guide and Tutorial,Francesco Chinello]

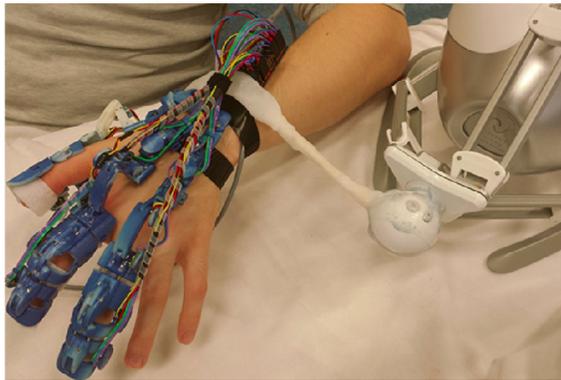
Overview:

- 1. Introduction of 3RPS parallel Manipulator robot*
- 2. Degrees of Freedom(DOF), links, joints*
- 3. Kinematics and dynamics analysis of 3RPS parallel manipulator robot*
- 4. Position Analysis of a 3 RPS Parallel Manipulator*
- 5. Jacobian Analysis for 3RPS Parallel Manipulator*
- 6. Dynamics analysis by experimental work by Matlab*
- 7. Conclusions and Future Research Directions*

1. Introduction of 3RPS parallel Manipulator robot

We are interested Parallel Manipulators robots due to:

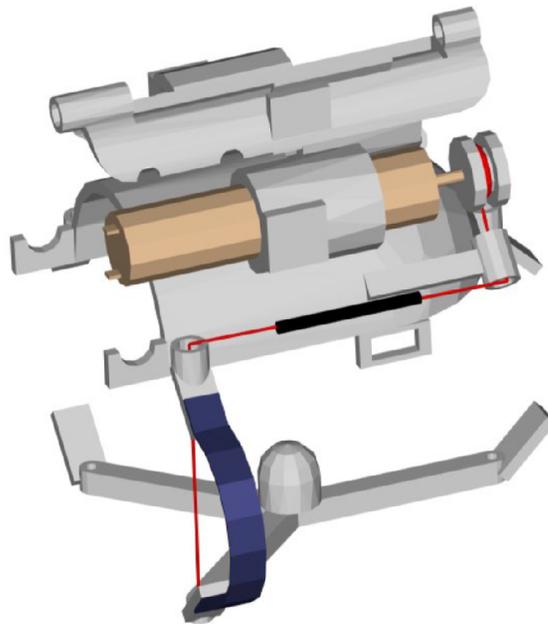
- In robotics we specify a manipulator as a device that we use to manipulate materials **without direct contact**



[Tactile feedback as a sensory subtraction technique in haptics for needle insertion, Dr. Francesco Chinello]

Introduction of 3RPS parallel Manipulator robot

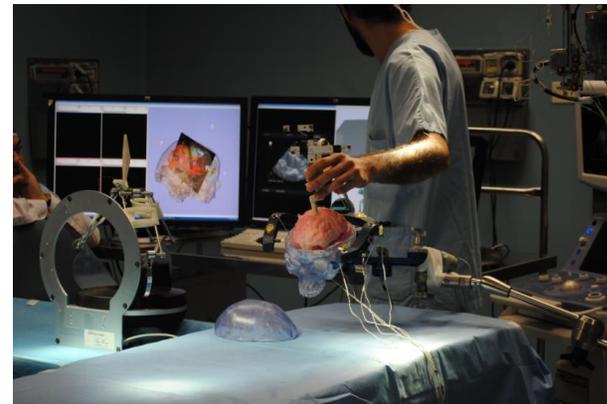
- *Dealing with radioactive or bio hazardous materials , teleoperation*
- *limited range of motion Explained in terms of DOF Degrees of Freedom(3)*



[Tactile feedback as a sensory subtraction technique in haptics for needle insertion, Dr. Francesco Chinello]

Introduction of 3RPS parallel Manipulator robot

- *Robotically assisted surgery ,in space and astronauts*

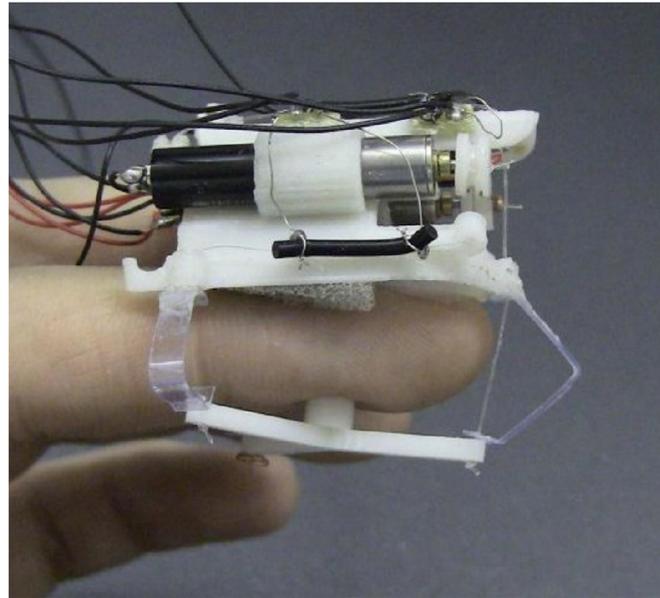


[Tactile feedback as a sensory subtraction technique in haptics for needle insertion, Dr. Francesco Chinello]

- *In industrial environments manipulator is a lift assist device for too heavy, too hot, too large lift maneuver*

Introduction of 3RPS parallel Manipulator robot

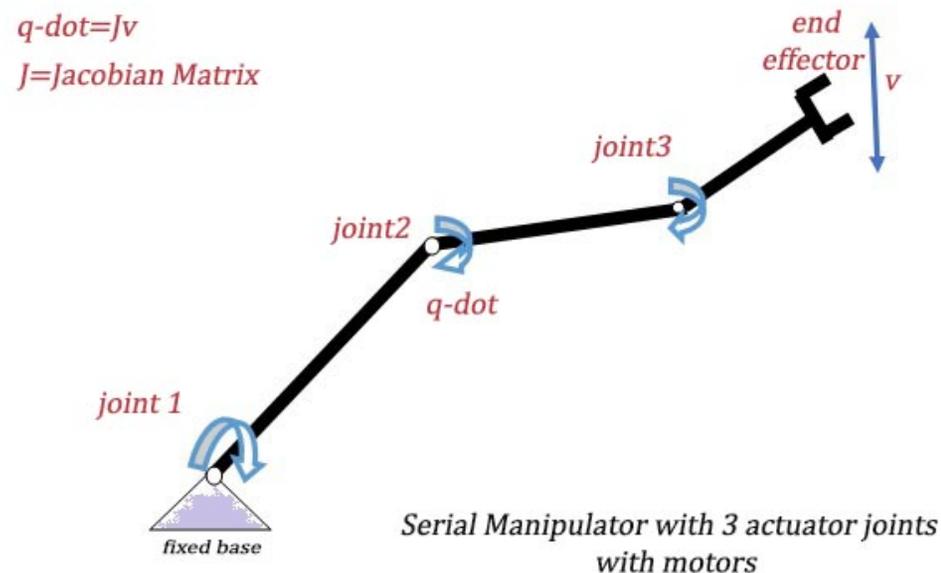
- Flexible design ,Manipulators configuration are composed of collections of links and joints combined by fixed and movable frames



[Tactile feedback as a sensory subtraction technique in haptics for needle insertion, Dr. Francesco Chinello]

Introduction of 3RPS parallel Manipulator robot

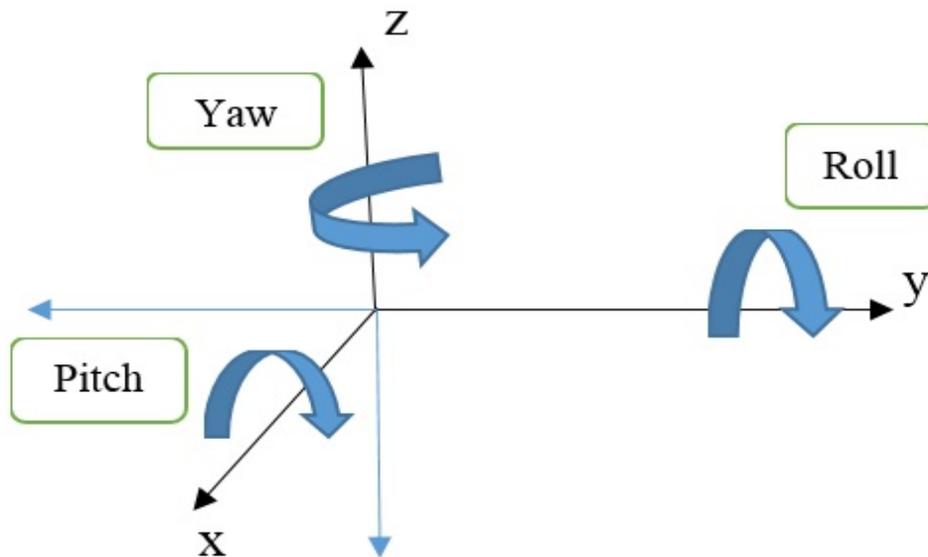
- *limited weight for the moving parts, move at a high speed*
- *high operational precision ,high positional accuracy within a limited workspace Capability of measuring forces and torques over the joints by using Jacobian matrix*



[Dynamics Analysis for a 3-PRS Spatial Parallel Manipulator-Wearable Haptic Thimble ,Masoud Moeini]

2. Degrees of Freedom(DOF), links, joints

Robot arms are described by their degrees of freedom and their spatial motion limitations



Six DOF:

forward/back(+y,-y),

up/down(+z,-z),

left/right(+x,-x)

pitch, yaw, roll

[Dynamics Analysis for a 3-PRS Spatial Parallel Manipulator-Wearable Haptic Thimble ,Masoud Moeini]

Degrees of Freedom(DOF), links, joints

We derive a general expression for the degrees of freedom of a mechanism in terms of the number of links, number of joints, and also types of joints incorporated in the mechanism

$$F = \lambda (n - j - 1) + \sum_{i=1}^j f_i$$

F : Overall degrees of freedom of a mechanism

f_i : Degrees of relative motion by joint i

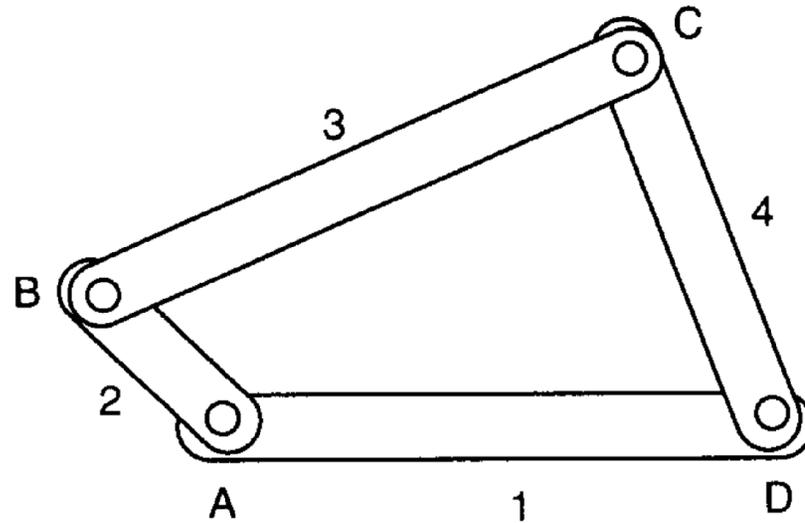
j : Number of joints in a mechanism

n : number of links in a mechanism, including the fixed link.

λ : degrees of freedom of each link in the space in which a mechanism is intended to function.

Degrees of Freedom(DOF), links, joints

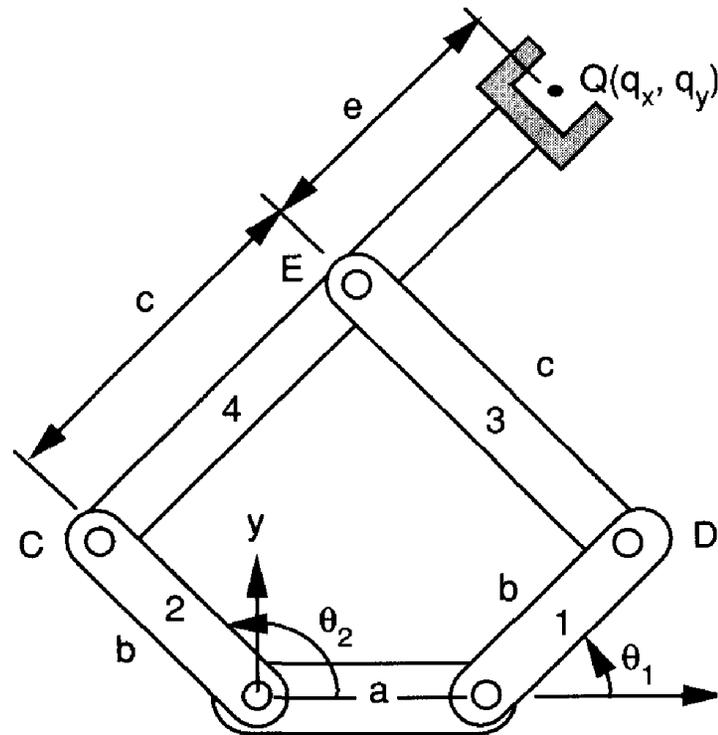
Example . Four Bar linkage .All the joints are revolute(dof1).We have $\lambda= 3$, $n = 4$, and $j = 4$. $F = 3(4 - 4 - 1) + 4 \times 1 = 1$.



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Degrees of Freedom(DOF), links, joints

Example 2. Five Bar Linkage .All the joints are revolute so we have $\lambda= 3$, $n = 5$, and $j = 5$. $F = 3(5 - 5 - 1) + 5 \times 1 = 2$.



[LUNGWENTSAI, " Robot Analysis The Mechanics of serial and Parallel Manipulators"]

Revolute joint (R), Prismatic joint (P) Spherical joint (S)

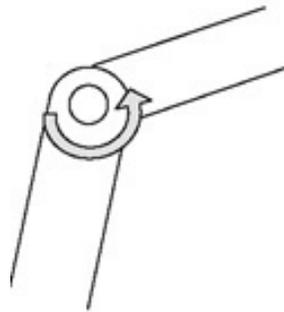
Revolute joint (R) Provides single axis rotation such as door hinges.

DOF(1)

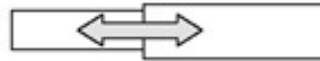
Prismatic joint (P) provides a linear sliding movement between two bodies, and is often called a slider. **DOF(1)**

Spherical joint (S) is a constraint element that allows the relative rotation of two bodies, It is sometimes referred to as a ball joint.

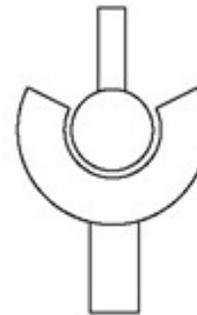
DOF(3)



Revolute
Joint



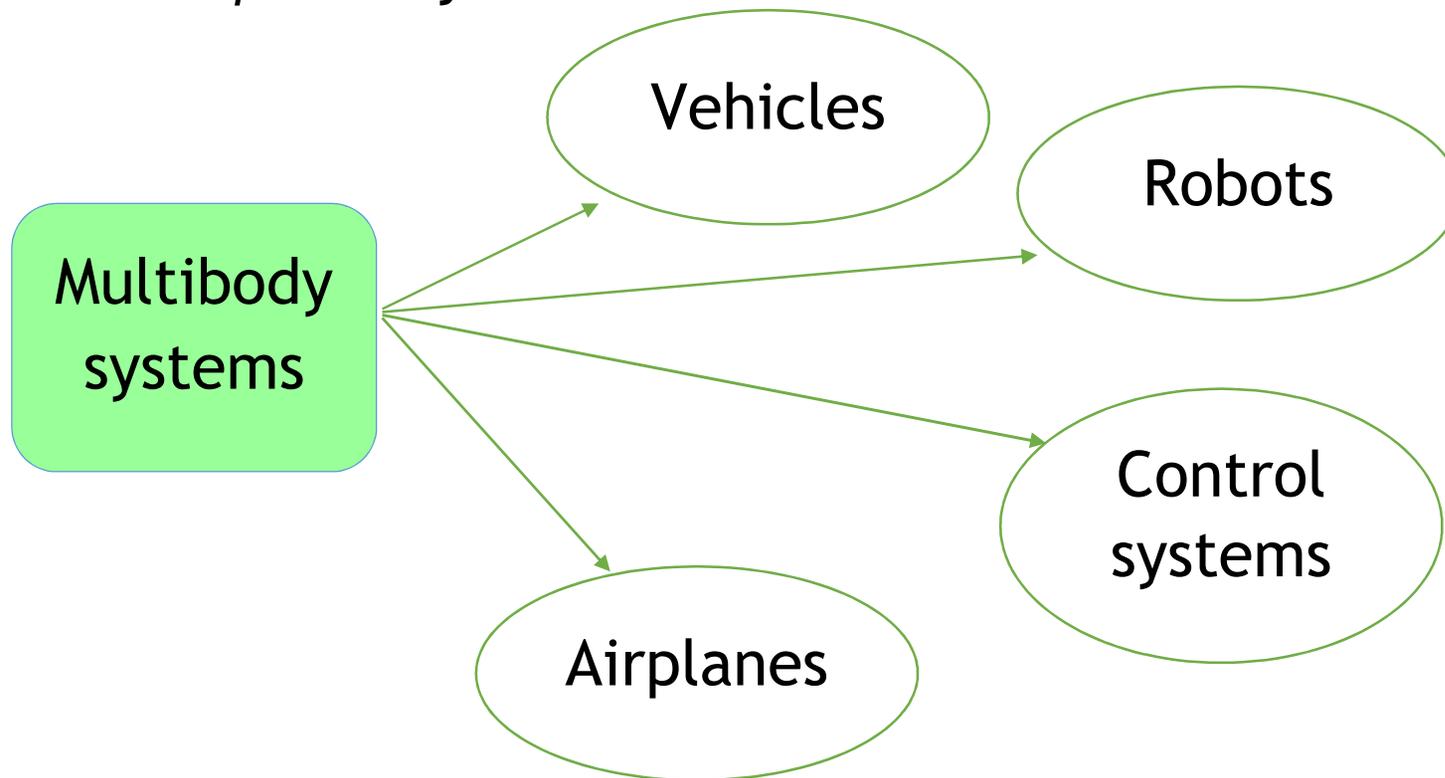
Prismatic
Joint



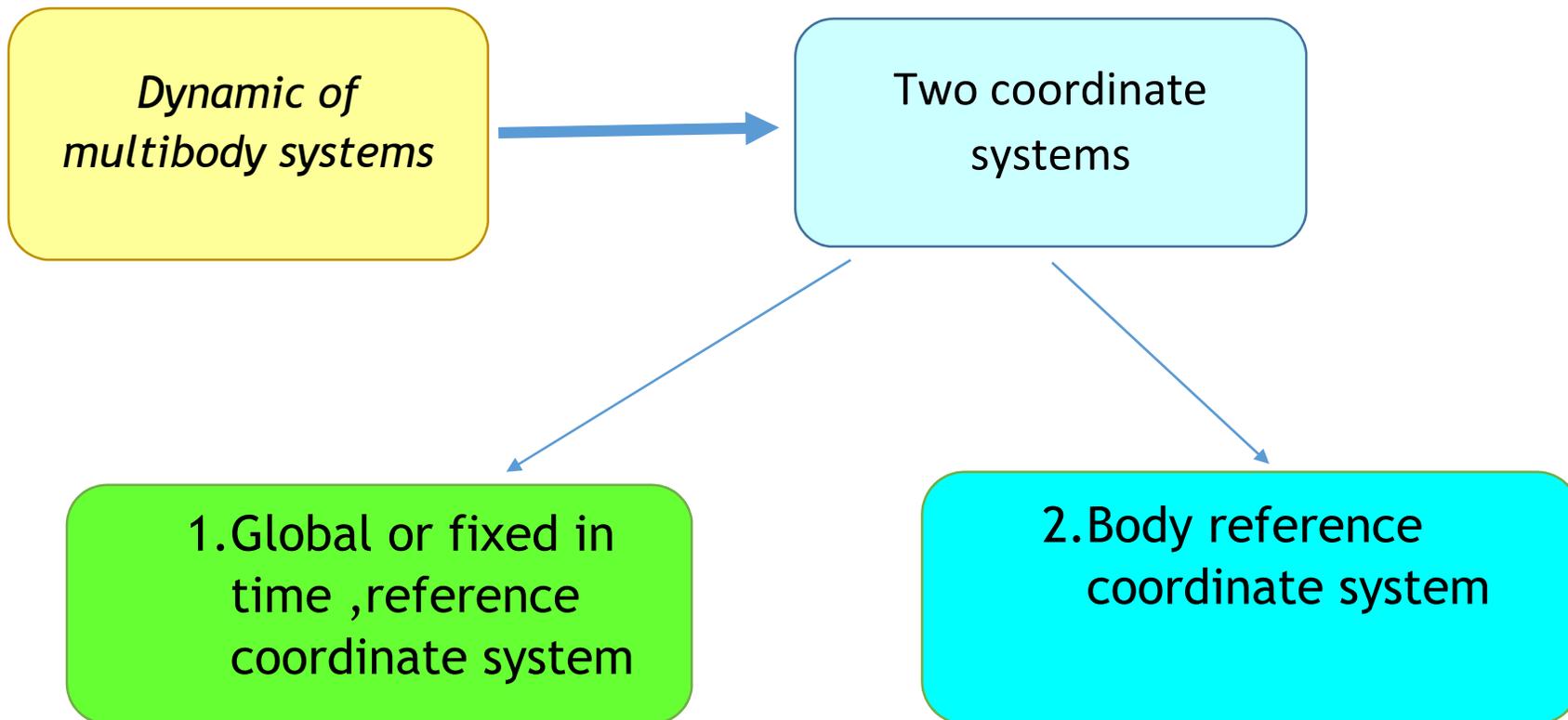
Spherical
Joint

4. Kinematics and dynamics analysis of 3RPS parallel manipulator robot

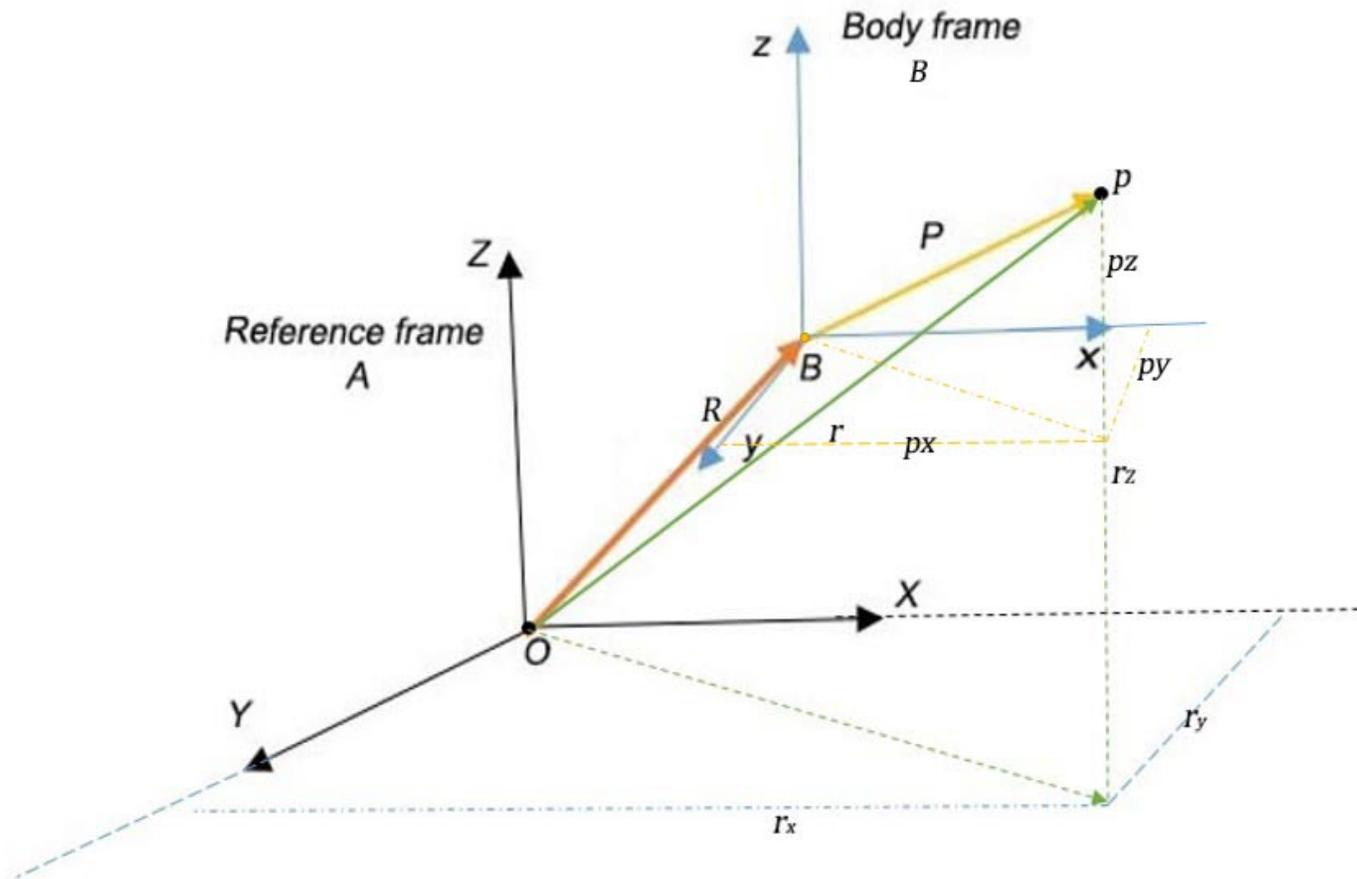
We consider 3RPS parallel manipulator in category of multibody or multicomponent systems



Generally, in dealing with multibody systems we need two coordinate system in order to describe the motion and displacement and rotations:



Kinematics and dynamics analysis of 3RPS parallel manipulator robot



[Dynamics Analysis for a 3-PRS Spatial Parallel Manipulator-Wearable Haptic Thimble ,Masoud Moeini]

$$r = R + P$$

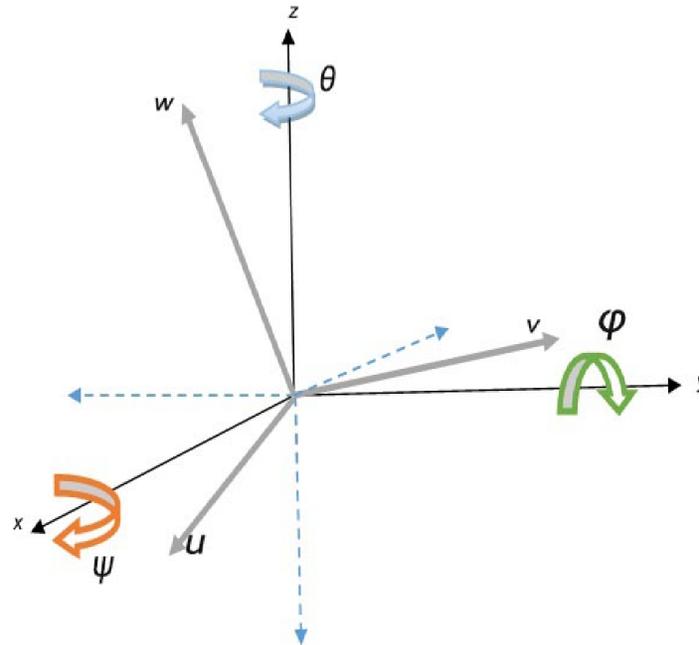
Kinematics and dynamics analysis of 3RPS parallel manipulator robot

$$\begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix} + \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

$$P' = {}^A R_B P ,$$

$$r = R + {}^A R_B P$$

Transformation matrix
derived from Euler's
theorem or direction
cosine theorem



[Dynamics Analysis for a 3-PRS Spatial Parallel Manipulator-Wearable Haptic Thimble ,Masoud Moeini]

Kinematics and dynamics analysis of 3RPS parallel manipulator robot

When a mobile platform performs rotation of ψ about x axes for our device we call it Roll transformation matrix.

$$R(x, \psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix}$$

When a mobile platform performs rotation of φ about y-axis for our device we call it Pitch transformation matrix.

$$R(y, \varphi) = \begin{bmatrix} c\varphi & 0 & s\varphi \\ 0 & 1 & 0 \\ -s\varphi & 0 & c\varphi \end{bmatrix}$$

When a mobile platform performs rotation of θ about z-axis for our device we call it Yaw transformation matrix.

$$R(z, \theta) = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Kinematics and dynamics analysis of 3RPS parallel manipulator robot

$${}^A R_B = R_{yaw} \times R_{pitch} \times R_{roll}$$

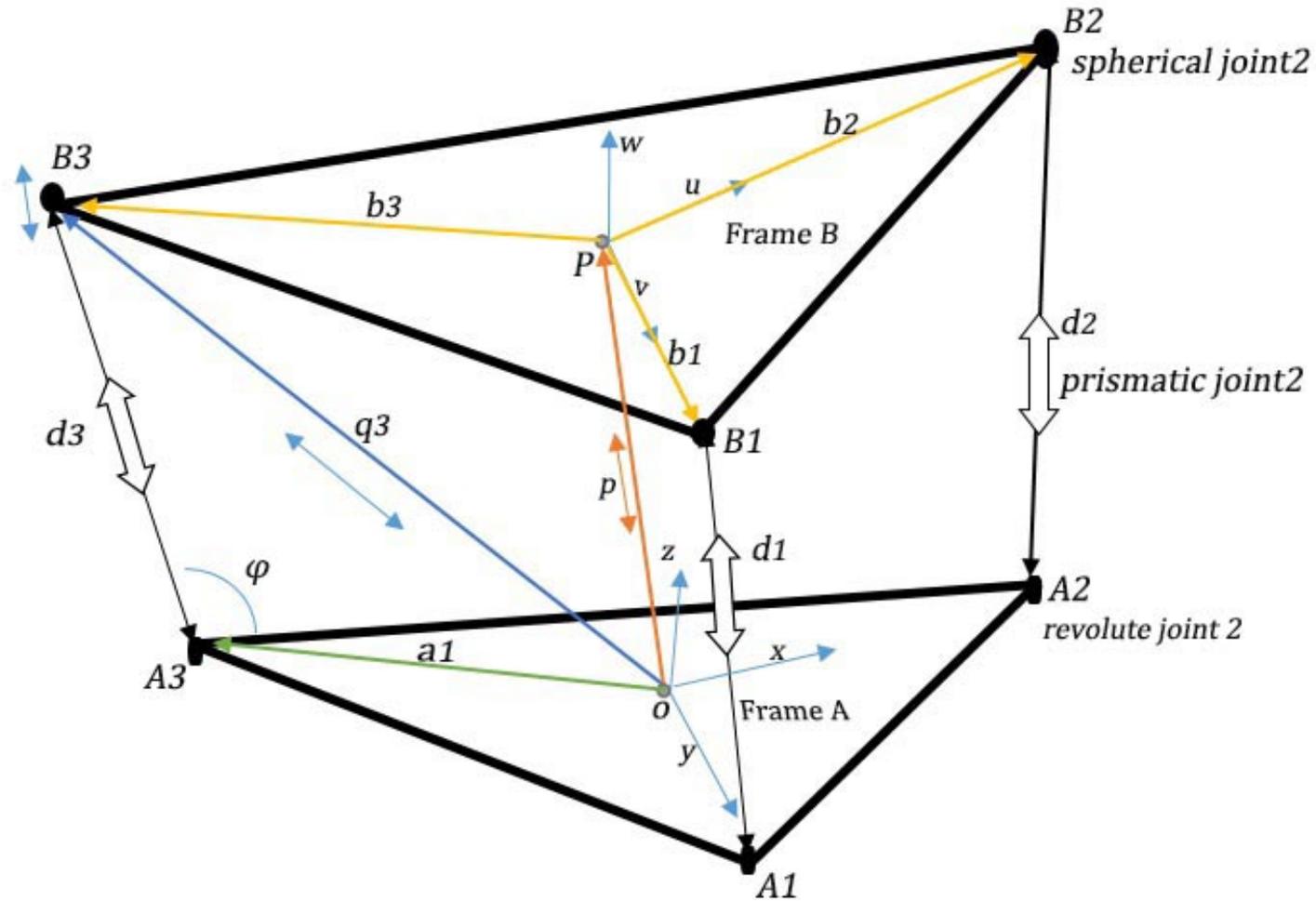
$${}^A R_B = R(z, \theta)R(y, \varphi)R(x, \psi) = \begin{bmatrix} c\theta c\phi + s\psi s\theta s\phi & -c\theta s\phi + s\psi s\theta c\phi & c\psi s\theta \\ c\psi s\phi & c\psi c\phi & -s\psi \\ -s\theta c\phi + s\psi c\theta s\phi & s\theta s\phi + s\psi c\theta c\phi & c\psi c\theta \end{bmatrix}$$

Velocity and acceleration :

$$\frac{d}{dt} (r = R + {}^A R_B P)$$

$$\frac{d^2}{dt^2} (r = R + {}^A R_B P)$$

Kinematics and dynamics analysis of 3RPS parallel manipulator robot



[Dynamics Analysis for a 3-PRS Spatial Parallel Manipulator-Wearable Haptic Thimble ,Masoud Moeini]

$$\mathbf{q}_i = \mathbf{p} + {}^A R_B {}^B \mathbf{b}_i$$

Kinematics and dynamics analysis of 3RPS parallel manipulator robot

- *We specified the relationships between the local and global components*
- *we obtained position in global reference system.*

5. Jacobian Analysis for 3RPS Parallel Manipulator

The three vectors of limb length can be expressed as

$$d_1 = [-c\varphi \ 0 \ -s\varphi]^T$$

$$d_2 = [c\varphi/2 \ -\sqrt{3}c\varphi/2 \ -s\varphi]^T$$

$$d_3 = [c\varphi/2 \ \sqrt{3}c\varphi/2 \ -s\varphi]^T$$

As we know:

$$a_1 = [a \ 0 \ 0]^T$$

$$a_2 = [-a/2 \ \sqrt{3}a/2 \ 0]^T$$

$$a_3 = [-a/2 \ -\sqrt{3}a/2 \ 0]^T$$

Jacobian Analysis for 3RPS Parallel Manipulator

Differentiating Eq. ($\mathbf{q}_i = \mathbf{p} + {}^A R_B {}^B \mathbf{b}_i$), with respect to time yields a velocity vector-loop equation as follow:

$$s_i \cdot v_p + (b_i \times s_i) \cdot \omega_p = \dot{d}_i s_i \cdot d_i \quad \text{for } i = 1, 2, 3$$

→ s_i is a unit vector pointing along $\overline{A_i B_i}$

- ✓ v_p is the three dimensional linear velocity of the moving frame B,
- ✓ ω_p is the angular velocity of the moving platform

By assuming :

$$\dot{X}_P = [v_p \ w_p]^T$$

Since DOF is 3 , the input vector for velocity of actuator joints can be written as

$$\dot{d} = \dot{q} = \dot{d}_i = [\dot{d}_1 \ \dot{d}_2 \ \dot{d}_3]^T, \quad J_x \dot{X}_P = J_q \dot{d}$$

Jacobian Analysis for 3RPS Parallel Manipulator

\dot{X}_P be vector of moving frame velocity

$$J_x = \begin{bmatrix} s_1^T & (b_1 \times s_1)^T \\ s_2^T & (b_2 \times s_2)^T \\ s_3^T & (b_3 \times s_3)^T \end{bmatrix}_{3 \times 6}$$

$$J_q = \begin{bmatrix} s_1 \cdot d_1 & 0 & 0 \\ 0 & s_2 \cdot d_2 & 0 \\ 0 & 0 & s_3 \cdot d_3 \end{bmatrix}_{3 \times 3}$$



$$\dot{q} = J \dot{X}_P$$

Where

$$J = J_q^{-1} J_x$$

Jacobian matrix, or simply Jacobian is defined as the matrix that we get the relationship between speed over the joint \dot{q} and the velocity of end effector \dot{x} or moving frame B

Jacobian Analysis for 3RPS Parallel Manipulator

$$\begin{cases} F_q = J^T_{ee}(q) F_x \\ J^T_{ee}(q) = J \end{cases} \longrightarrow \boxed{F_q = J F_x}$$

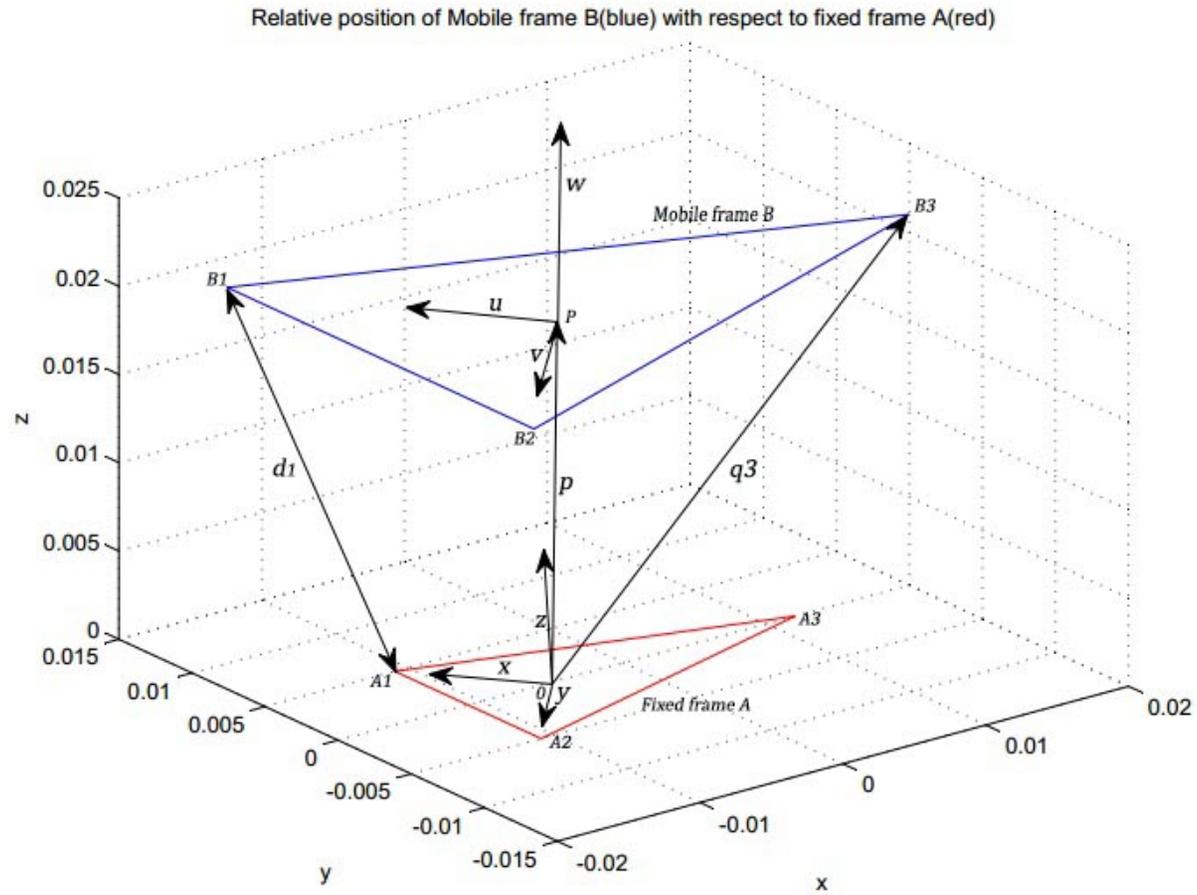
- ✓ F_x the force applied to the hand or slave robot finger
- ✓ F_q the forces in joint-space

6. Dynamics analysis by experimental work by Matlab

a1,a2,a3	b1,b2,b3	Ψ, θ, ϕ	$p = [p_x p_y p_z]$	transformation matrix ${}^A R_B$	$q_1, q_2, q_3 =$ $\overline{OB_i}$	$\dot{X}_p = [v_p w_p]^T$
a1 = 0.0120 0 0	b1 = 0.0200 0 0	psi = 0 theta = 0 phi = 0	p = 0 0 0.0210	R = 1 0 0 0 1 0 0 0 1	q1 = 0.0200 0 0.0210 q2 = -0.0170 -0.0105 0.0210 q3 = -0.0170 0.0105 0.0210	x_dot = 1.0e-03 * 0 0 1.0000 0 0
$d = [d_1 d_2 d_3]$			Jacob =			
d = -1.0000 1.0000 1.0000 0 -1.7321 1.7321 0 0 0			0.3560 0 0.9345 0 -0.0112 0 -0.2711 -0.2439 0.9312 -0.0047 0.0102 0.0013 -0.2711 0.2439 0.9312 0.0047 0.0102 -0.0013			
Velocity over the joints $\dot{X}_p = J\dot{q}$, \dot{q} Matrix 3 x 1			F_x	$F_q = JF_x$		
q_dot = 1.0e-03 * 0.2441 0.2441 0.2441			$F_x =$ 0 0 1 0 0 0	$F_q =$ 0.9345 0.9312 0.9312		

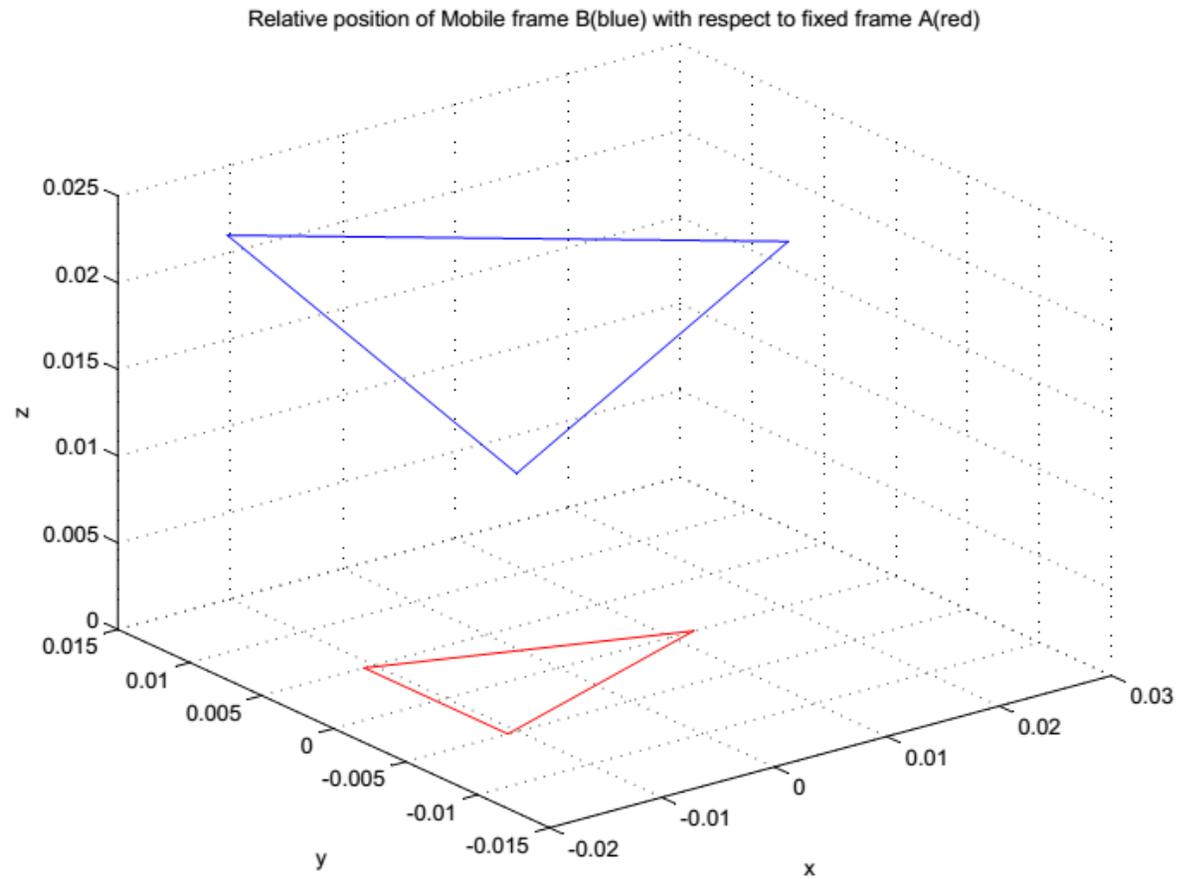
[Dynamics Analysis for a 3-PRS Spatial Parallel Manipulator-Wearable Haptic Thimble ,Masoud Moeini]

Dynamics analysis by experimental work by Matlab



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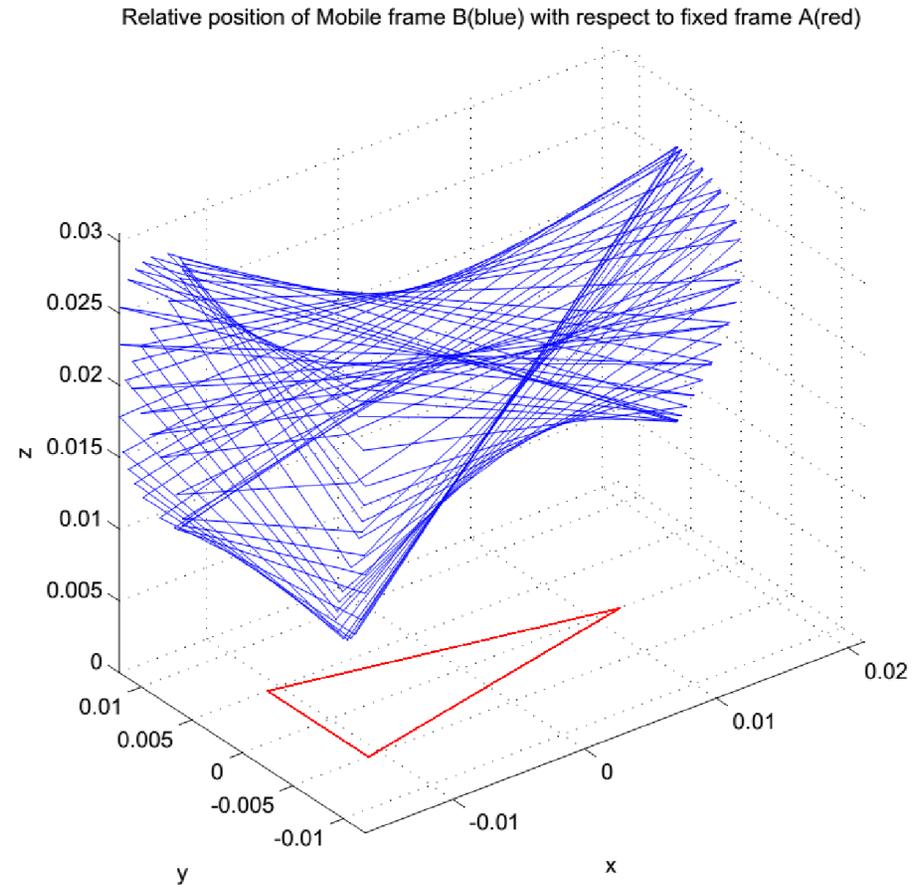
Dynamics analysis by experimental work by Matlab

a1,a2,a3	b1,b2,b3	Ψ, θ, ϕ	p = $[p_x p_y p_z]$	transformation matrix ${}^A R_B$	q1, q2, q3 = $\overline{O B_i}$	$\dot{X}_P = [v_p w_p]^T$
a1 = 0.0120 0 0	b1 = 0.0200 0 0	psi = 0.2932	p = 0.0004 0 0.0210	R = 1.0000 0 0 0 0.9573 -0.2890 0 0.2890 0.9573	q1 = 0.0204 0 0.0210 q2 = -0.0166 -0.0101 0.0180 q3 = -0.0166 0.0101 0.0240	x_dot = 1.0e-03 * 0 1.0000 0 0
a2 = -0.0109 -0.0050 0	b2 = -0.0170 -0.0105 0	theta = 0 phi = 0				
a3 = -0.0109 0.0050 0	b3 = -0.0170 0.0105 0					
$d = [d_1 d_2 d_3]$			Jacob =			
d = -1.0000 1.0000 1.0000 0 -1.7321 1.7321 0 0 0			0.3724 0 0.9281 0 -0.0111 0 -0.2915 -0.2589 0.9209 -0.0046 0.0100 0.0014 -0.2256 0.2004 0.9534 0.0048 0.0104 -0.0011			
Velocity over the joints $\dot{X}_p = J\dot{q}$, \dot{q} Matrix 3×1		F_x	$F_q = JF_x$			
q_dot = 1.0e+13 * -0.0735 3.0247 -2.2518		Fx = 0 0 1 0 0 0	Fq = 0.9281 0.9209 0.9534			

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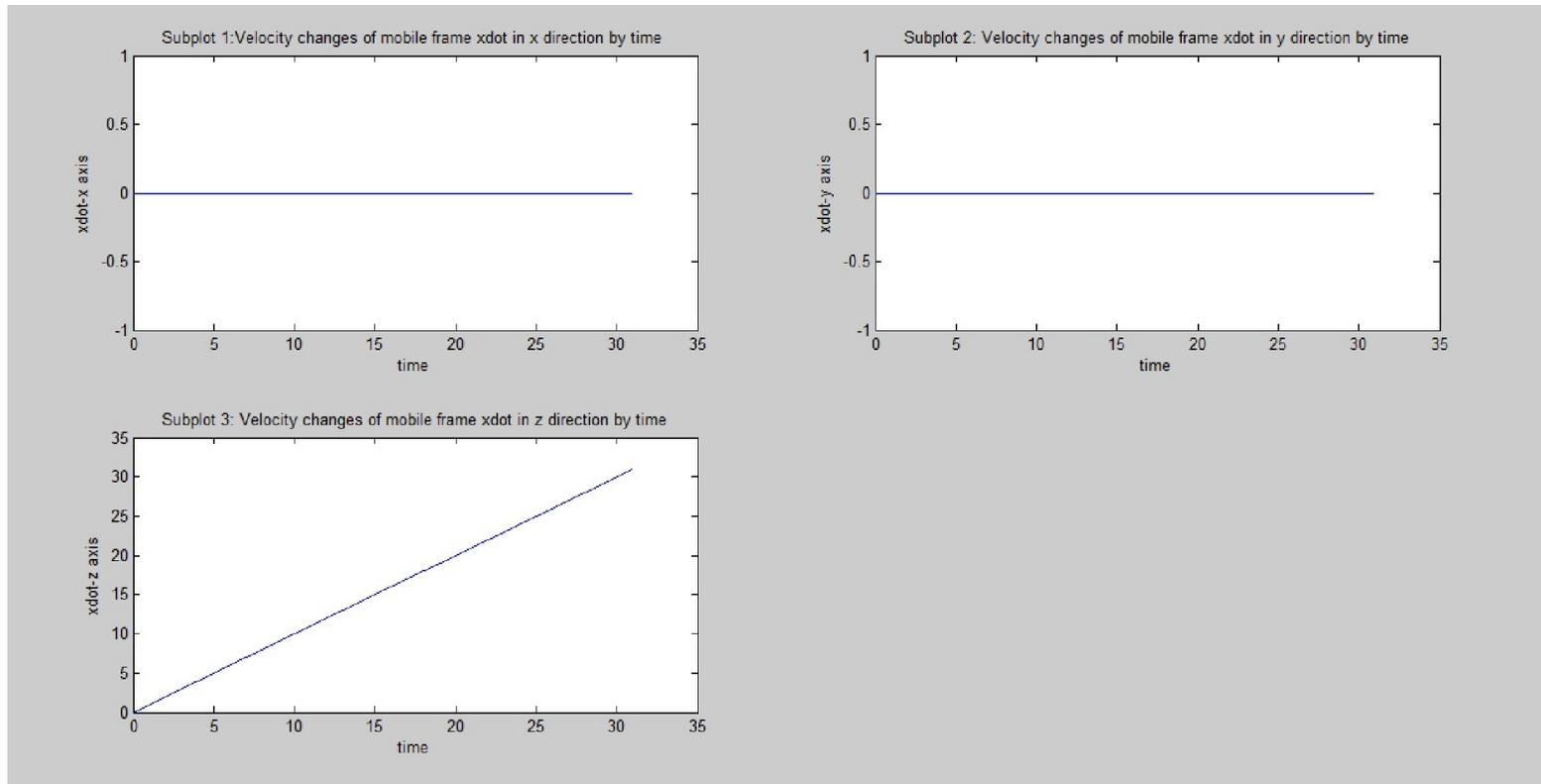
Dynamics analysis by experimental work by Matlab

$$\begin{aligned}\psi &= 0.5 \cdot \sin(\alpha) \\ \theta &= 0.5 \cdot \cos(\alpha) \\ \alpha &= [0..30] \\ \dot{X} &\text{ is } [0 \ 0 \ v \ 0 \ 0 \ 0]. \\ v &= [0..30]\end{aligned}$$



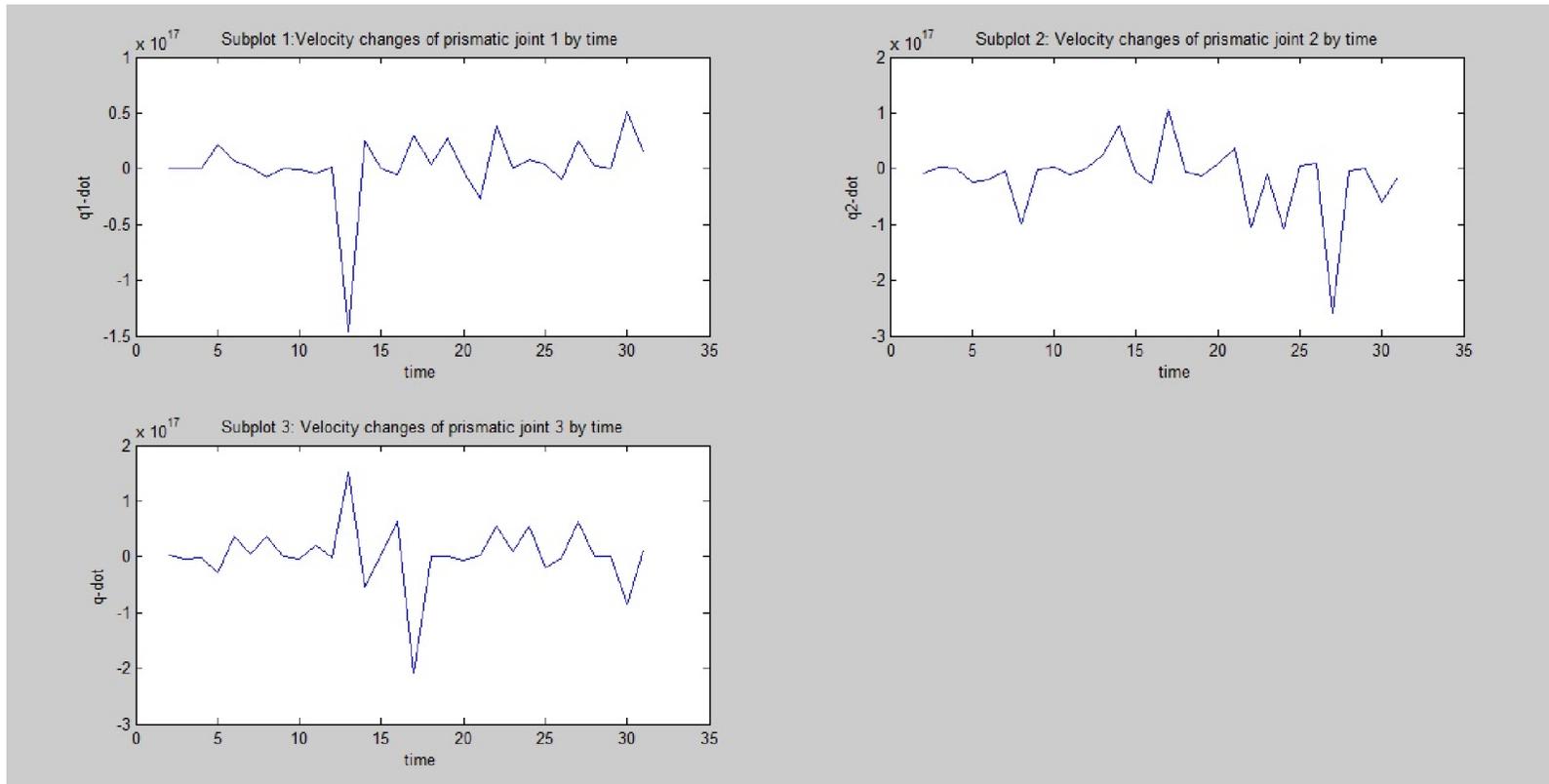
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Dynamics analysis by experimental work by Matlab



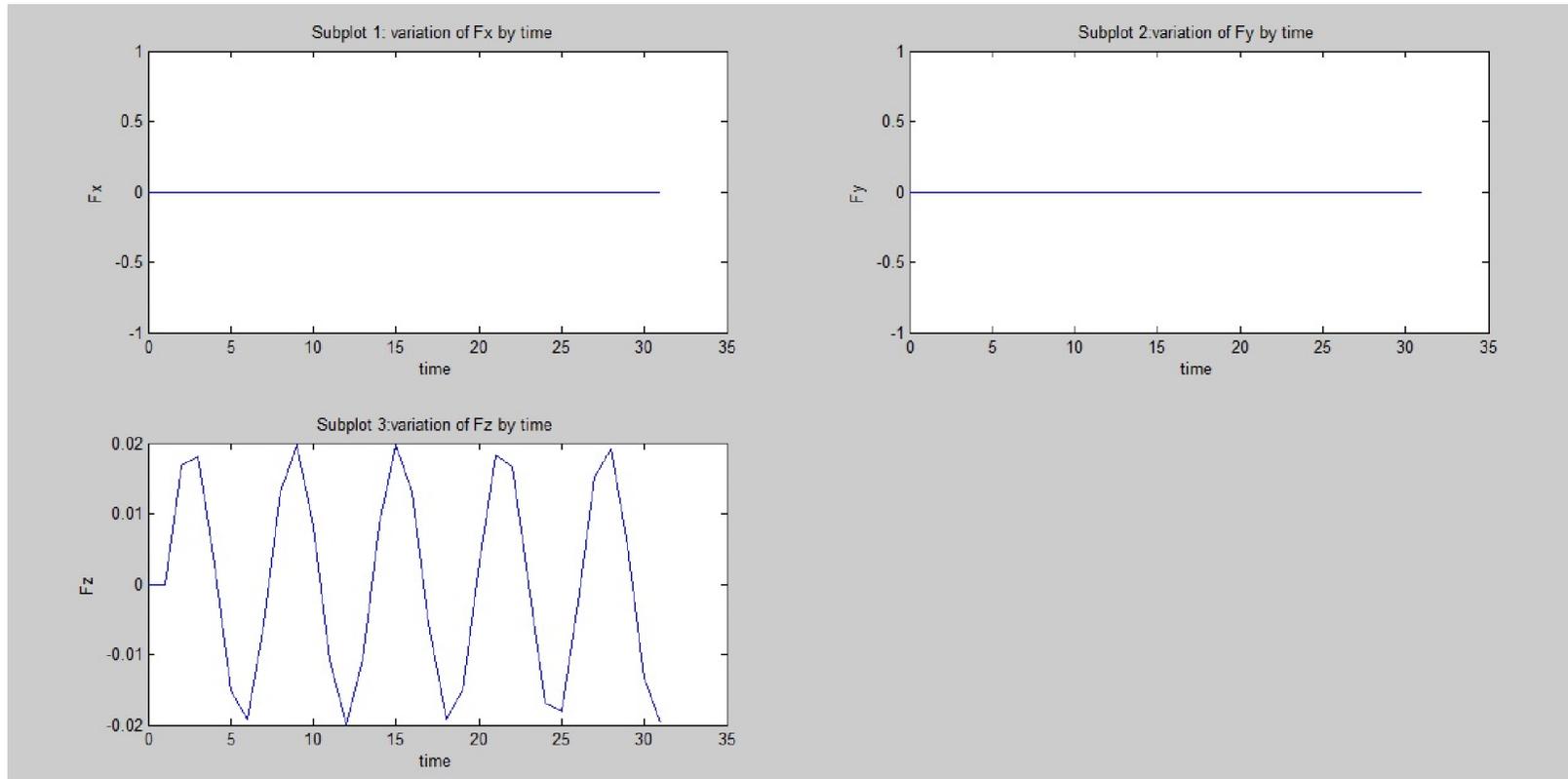
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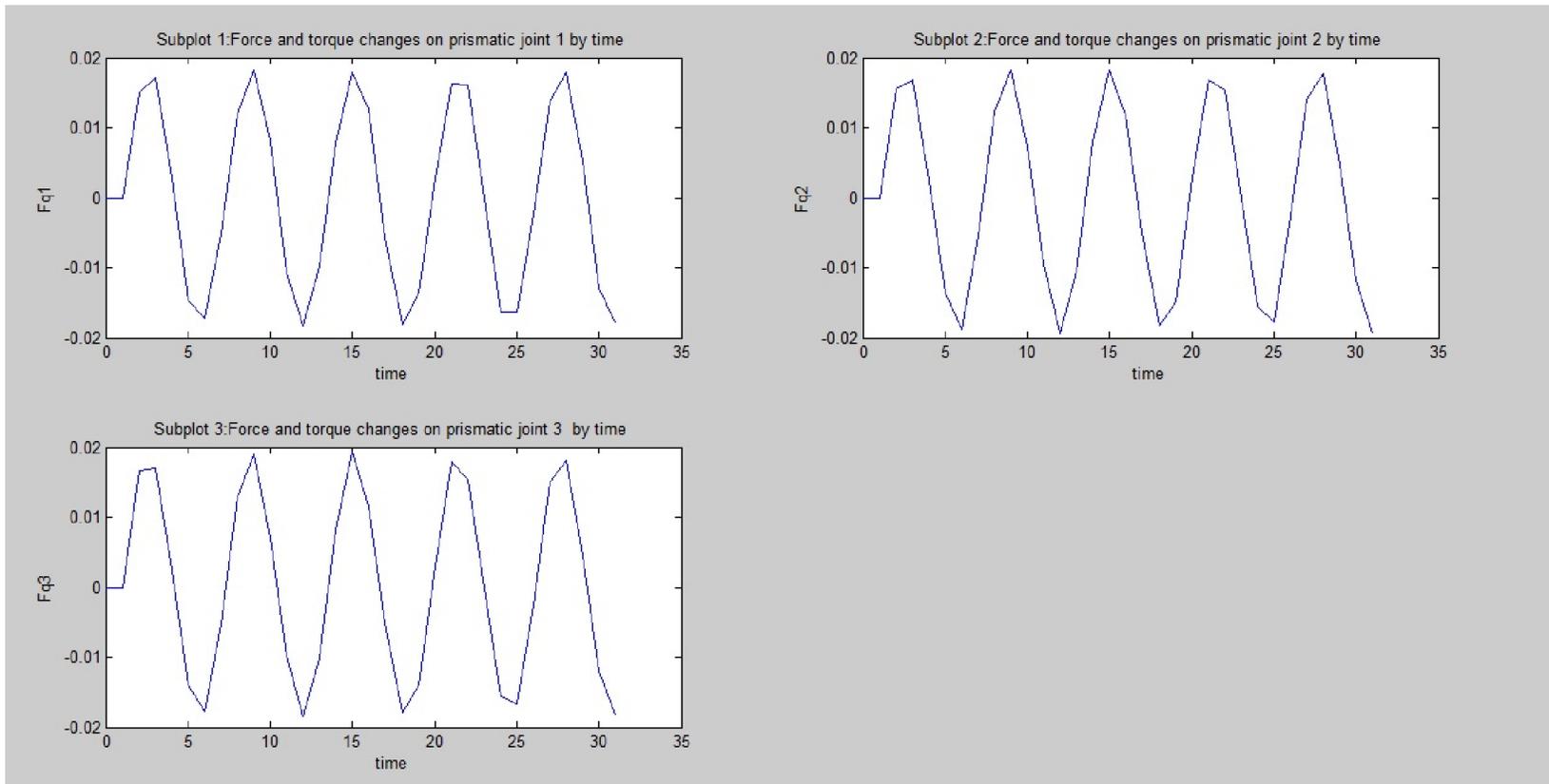
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7. Conclusions and Future Research Directions

- ✓ We induced completely the kinematics of multibody systems generally and particularly the best mathematics model description for purposed 3RPS parallel manipulator wearable haptic thimble robot
- ✓ Try to improve performance and functionality in order to analysis position, velocity of moving frame B
- ✓ computing forces, torques over the joints in order to achieve high level control performance over this device

Conclusions and Future Research Directions

- ✓ Concept of Jacobian ,the relationship between speed over the joint and the velocity of end effector or moving frame
- ✓ This effort can also be applied to other parallel mechanisms with different DOF.

References:

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