Introduction to Robotics

Assignment #5

Due: 28.06.2015, 23.59

Preferred tools/languages for programming, plotting and solving are

`python`, `matlab`, `octave` or `gnuplot`.

Always provide source code as well as all intermediate steps. Make sure to annotate your code in detail in order to provide readability and fast understanding. `"Here, some magic happens"` will cost you points.

Task 5.1 (5 points) Basis-Splines - Programming Task: Draw the basis splines of the order 1 to 4 within the intervals $[t_i, t_{i+1}], i = 0, 1, 2, 3, 4$. Provide functions and plot within your solution as well as the attached source code.

Task 5.2 (4 points) Basis-Splines - Direct Computing: In the lecture the recursive calculation of the basis splines has been introduced. With regard to real-time time critical applications, direct computation for a given order has advantages. Deduce the explicit formula for the basis splines with the order 1 to 3.

Task 5.3 (5 points) Lagrange polynomial: Given a set of data points $(0, 1), (1, 3), (3, -2), (5, 4)$. Calculate the Lagrange polynomial $p_3(x)$ through the data points.

Simplify the equation as much as possible. Use a tool of your choice (`gnuplot` is preferred) to visualize the four Lagrange basis polynomials and the polynomial $p_3(x)$.

Task 5.4 (6 points) PID-controller: Consider a DC motor with

J moment of inertia of the rotor $= 0.1\ kg\,m^2$

b motor viscous friction constant $= 0.1\ N\,m\,s$

K motor torque constant $= 0.01\ N\,m/Amp$

R electric resistance $= 1\ Ohm$

L electric inductance $= 0.5\ H$

Calculate the response of a PID-controller with the factors k_p, k_i and k_d for the DC motor[1] with a 1-rad/sec step reference and plot the function for both substasks with

1. $k_p = 20$, $k_i = 0$, $k_d = 0$,
2. $k_p = 500$, $k_i = 0$, $k_d = 0$ and
3. $k_p = 100$, $k_i = 50$, $k_d = 0$

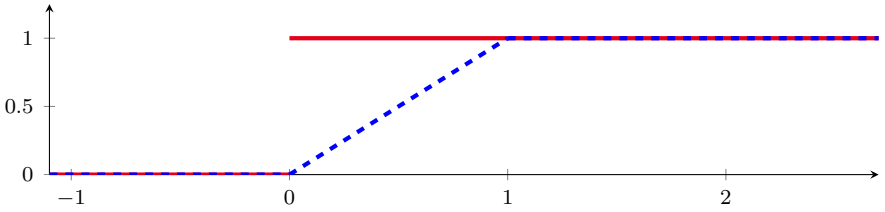
using a tool of your choice (`matlab` should be preferred).

5.4.1 (3 points): to the step function:

$$\mathbf{y(t)} = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

5.4.2 (3 points): the ramp function

$$\mathbf{y(t)} = \begin{cases} 0, & t < 0 \\ t, & 0 \leq t < 1 \\ 1, & t \geq 1 \end{cases}$$



5.4.3 *Bonus* (4 points): find k_p, k_i, k_d for the step function such that

- settling time is less than 2 seconds
- overshoot is less than 5%
- steady-state error less than 1%

[1] Find the transfer function of a DC motor to be $\frac{K}{(J \cdot s + b) \cdot (L \cdot s + R) + K^2}$