

# A Hybrid Genetic Swarm Algorithm for Interactive Inverse Kinematics

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Master Thesis

Colloquium

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# Introduction and Motivation

## Problem Statement

*How to adjust a set of joints in order to move an end effector to reach a Cartesian configuration of position and/or orientation?*

## Kinematics

„Kinema“ = „Movement / Motion“

→ Field of classical mechanics

→ Motion of rigid bodies by position, velocity, acceleration

→ **No** consideration of physical dynamics (mass, force, torque, ...)



# Introduction and Motivation

## Applications

### **Robotics**

- *Grasping and Object Manipulation*
- *Bi-Pedal and Multi-Pedal Walking*
  - *Human Interaction*
  - *Manufacturing*

### **Games Industry**

- *Believable characters*
  - *Realistic motion*
- *Dynamic and flexible animations*

### **Film Industry**

- *Motion Tracking*



# Introduction and Motivation

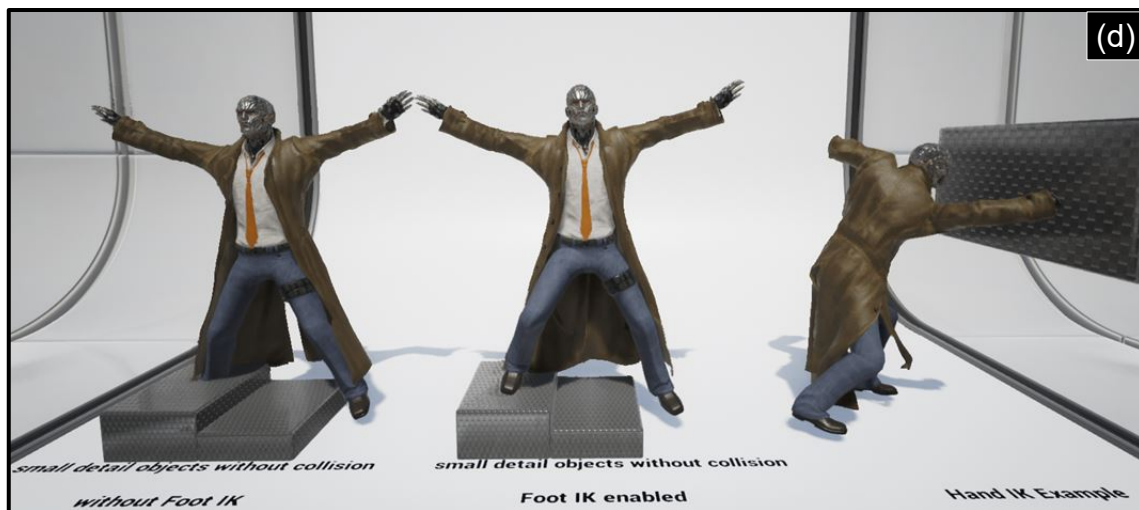
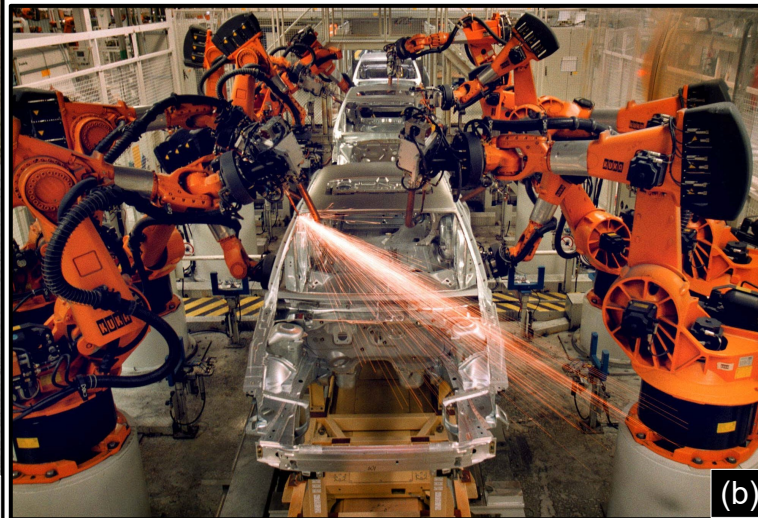
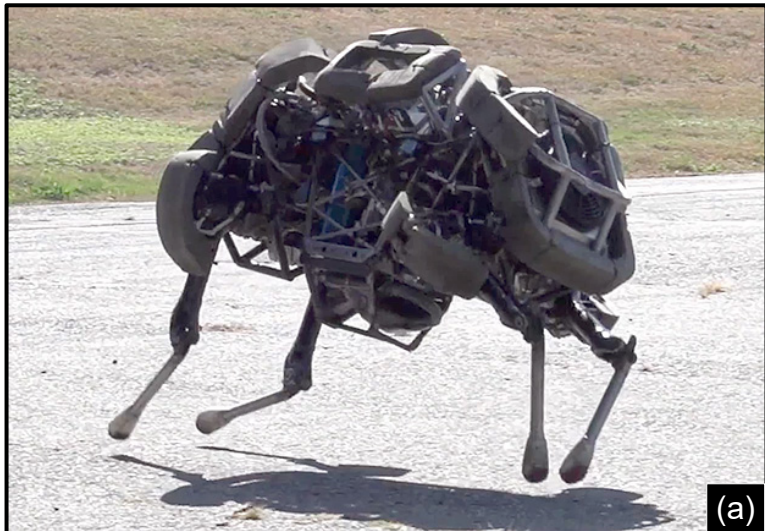
## Challenges

- Zero up to infinite solutions
  - Geometric complexity
  - High dimensionality
- Suboptimal extrema and singularities
  - Joint constraints and types
  - Solution quality
- *Accuracy versus Computation Time*
  - *Robustness and Reliability*
- Displacement between solutions
  - Self-Collision

...



# Introduction and Motivation



# Introduction and Motivation

## Major goals of this thesis

A universal IK solver for arbitrary kinematic chains

- Real-time capability for interactive frame rates
- High accuracy for both position and orientation
- Flexible, few parameters and easy-to-use

Novel algorithmic improvements for biologically-inspired optimization strategies

- Modular extensions applicable for various problems
- Higher adaptivity in exploitation and exploration
- Biologically-plausible evolutionary concepts

# Problem Formalization

$\mathcal{X}$  → Cartesian configuration of position and/or orientation  
 $\theta$  → Joint variable configuration

## Forward Kinematics (FK)

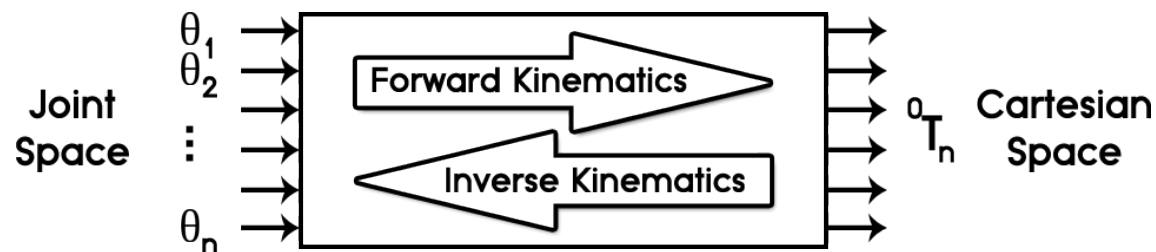
$$\mathcal{X} = f(\theta)$$

- Straightforward computation
- Unique solution
- Only requires kinematic specifications and joint values

## Inverse Kinematics (IK)

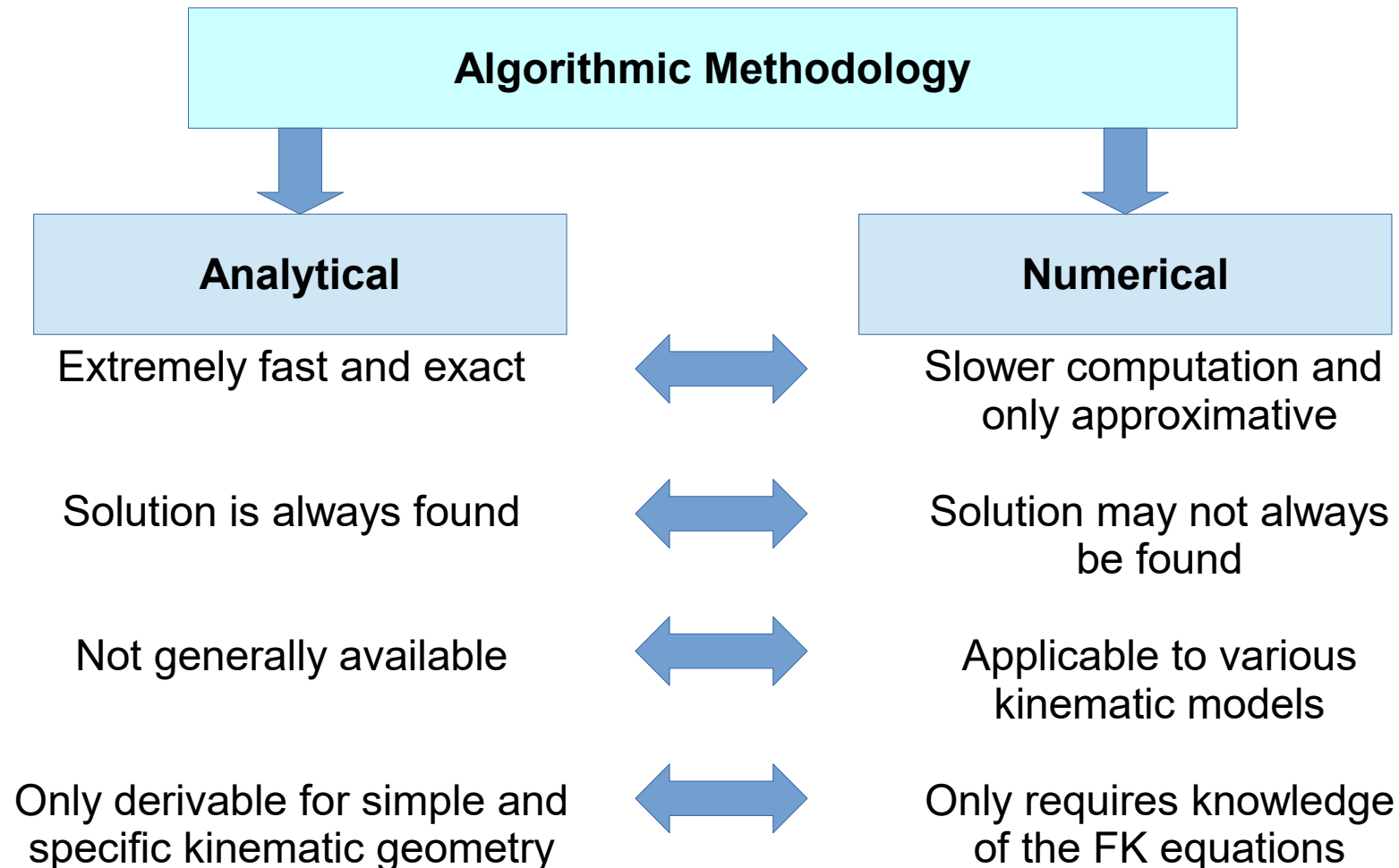
$$\theta = f^{-1}(\mathcal{X})$$

- Highly non-trivial
- Complexity scales rapidly
- Analytical versus Numerical





# Problem Formalization



# Problem Formalization

$Y$  → Cartesian target of position and/or orientation  
 $\mu \Theta$  → Weighted joint variable change

Numerical IK Update

$$\theta' = \theta + \mu \Theta \quad d(f(\theta'), \mathcal{Y}) < d(f(\theta), \mathcal{Y})$$

Pose Distance

$$d_p = w_t \hat{d}_t + w_r d_r$$

(Rebalanced) Translational Distance

$$\hat{d}_t = \frac{\pi d_t}{\sqrt{l \Delta}} \quad d_t = \|p_1 - p_2\|$$

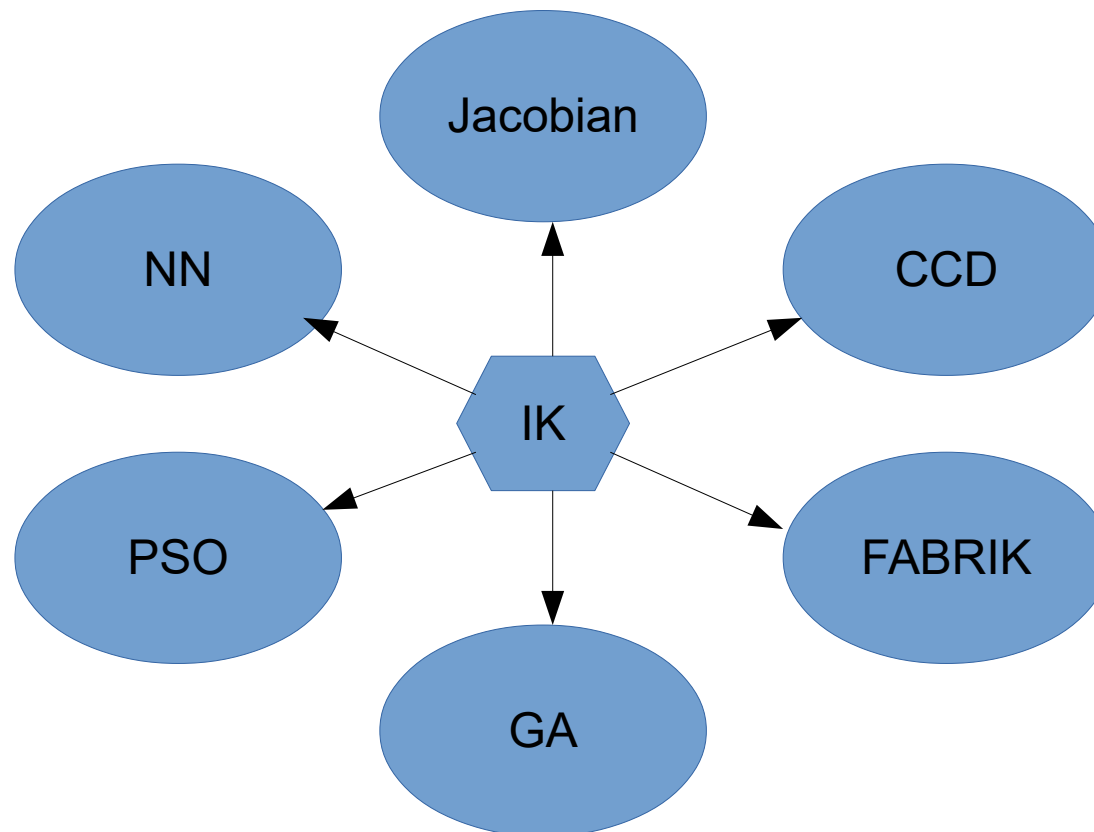
Rotational Distance

$$d_r = q_1 \cdot q_2$$

$l$  → Length of the kinematic chain  
 $\Delta$  → Distance from base to end effector

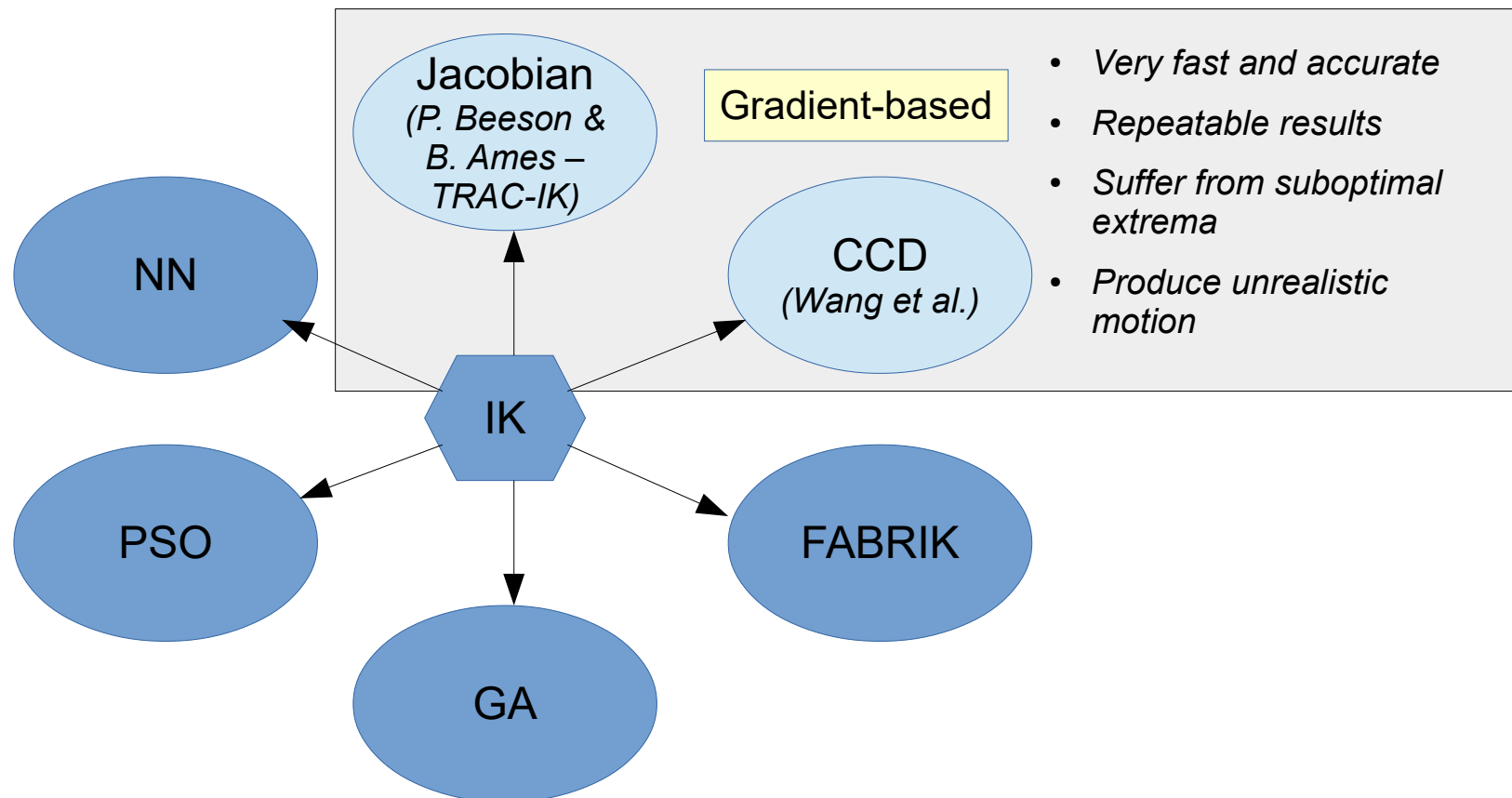
# Related Work

- IK researched over decades
- Very many different approaches with focus on numerical



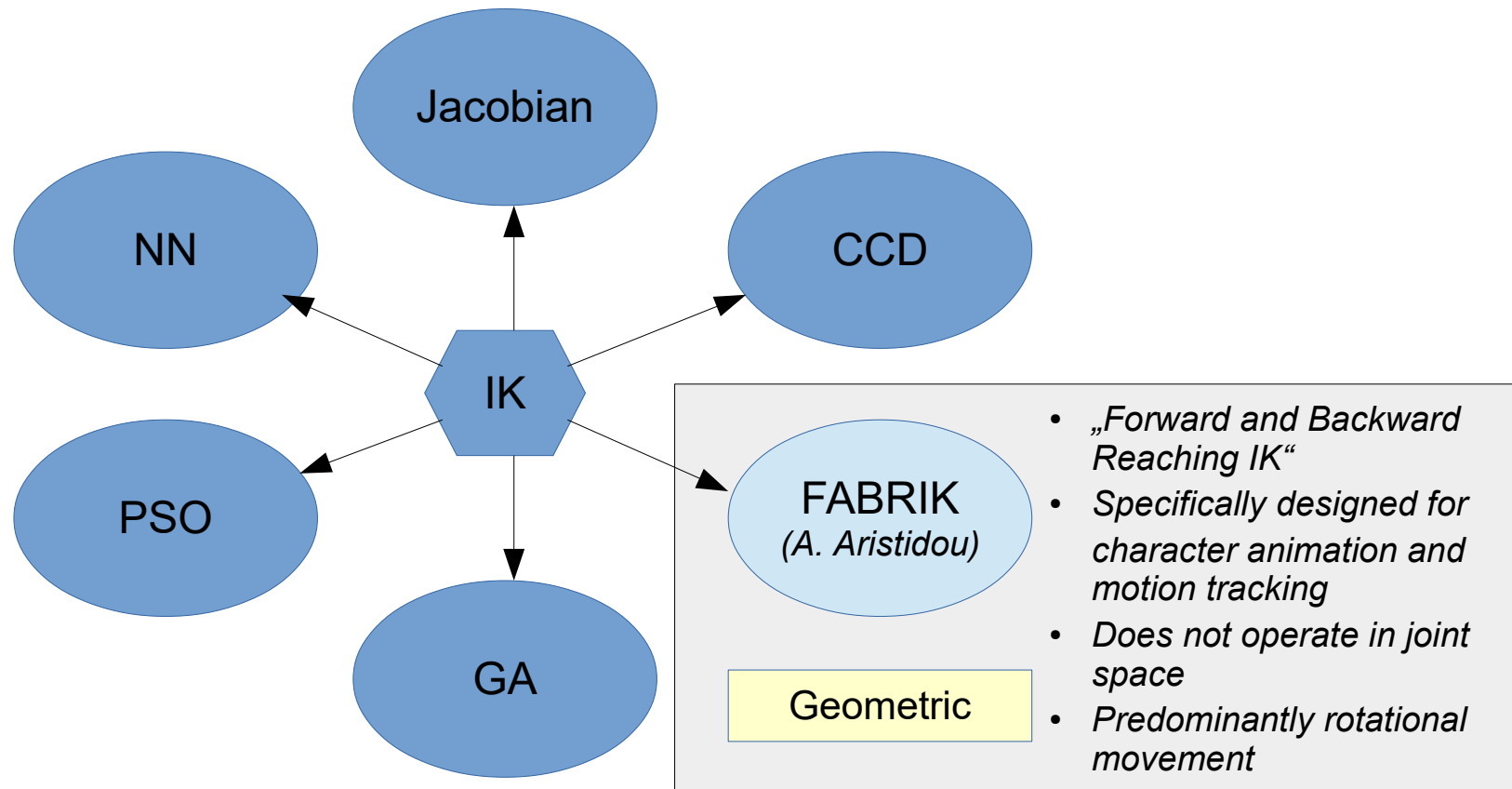
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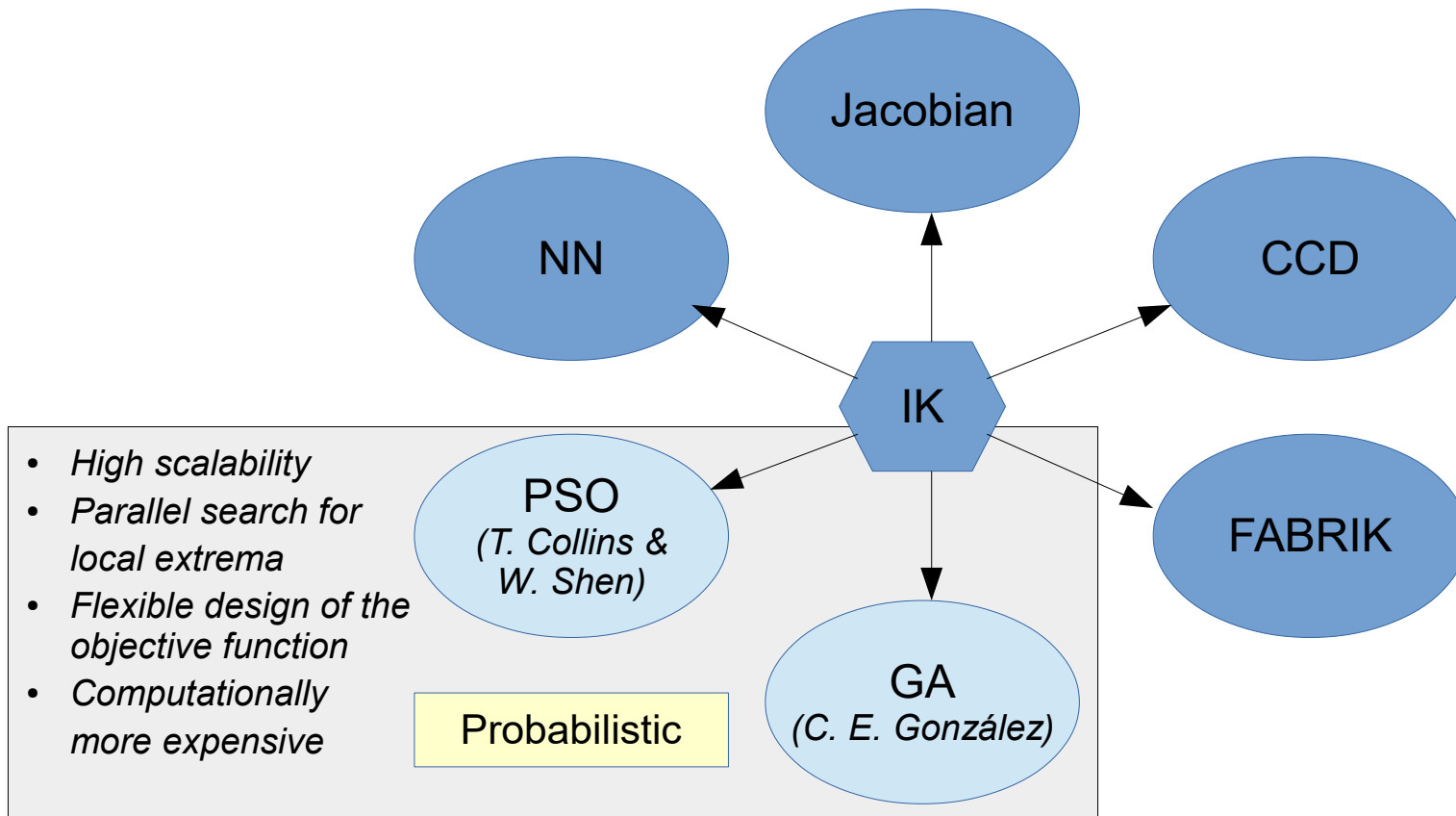
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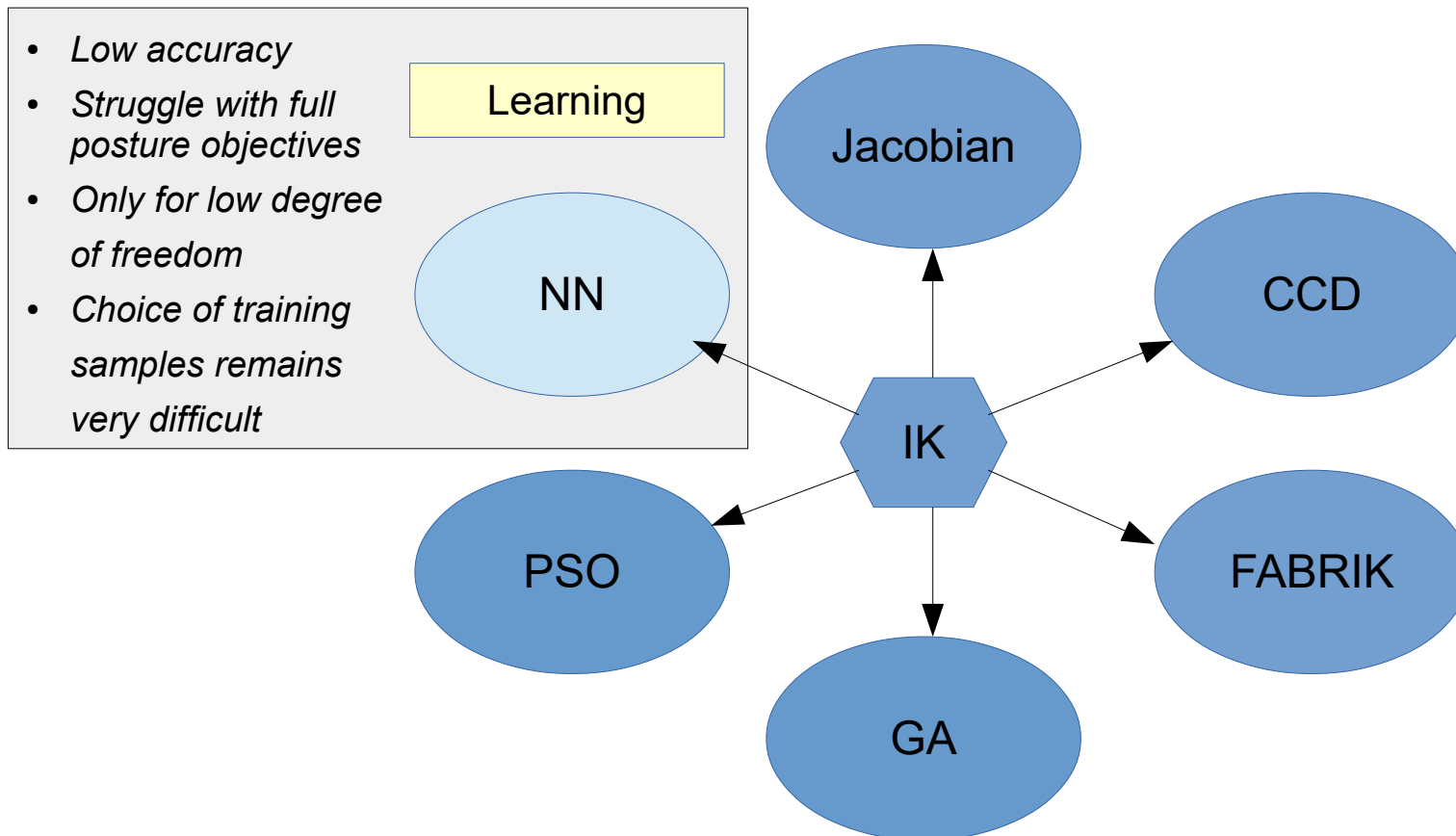


- *High scalability*
- *Parallel search for local extrema*
- *Flexible design of the objective function*
- *Computationally more expensive*

Probabilistic

# Related Work

- IK researched over decades
- Very many different approaches with focus on numerical



# Algorithmic Approach

## Design Objectives

Success

*A solution that is existent shall be found.*

Accuracy

*The solution shall be as precise as possible.  
→ approximately or less than 1mm*

Time

*The solution shall be found as fast as possible.  
→ few ms*

Displacement

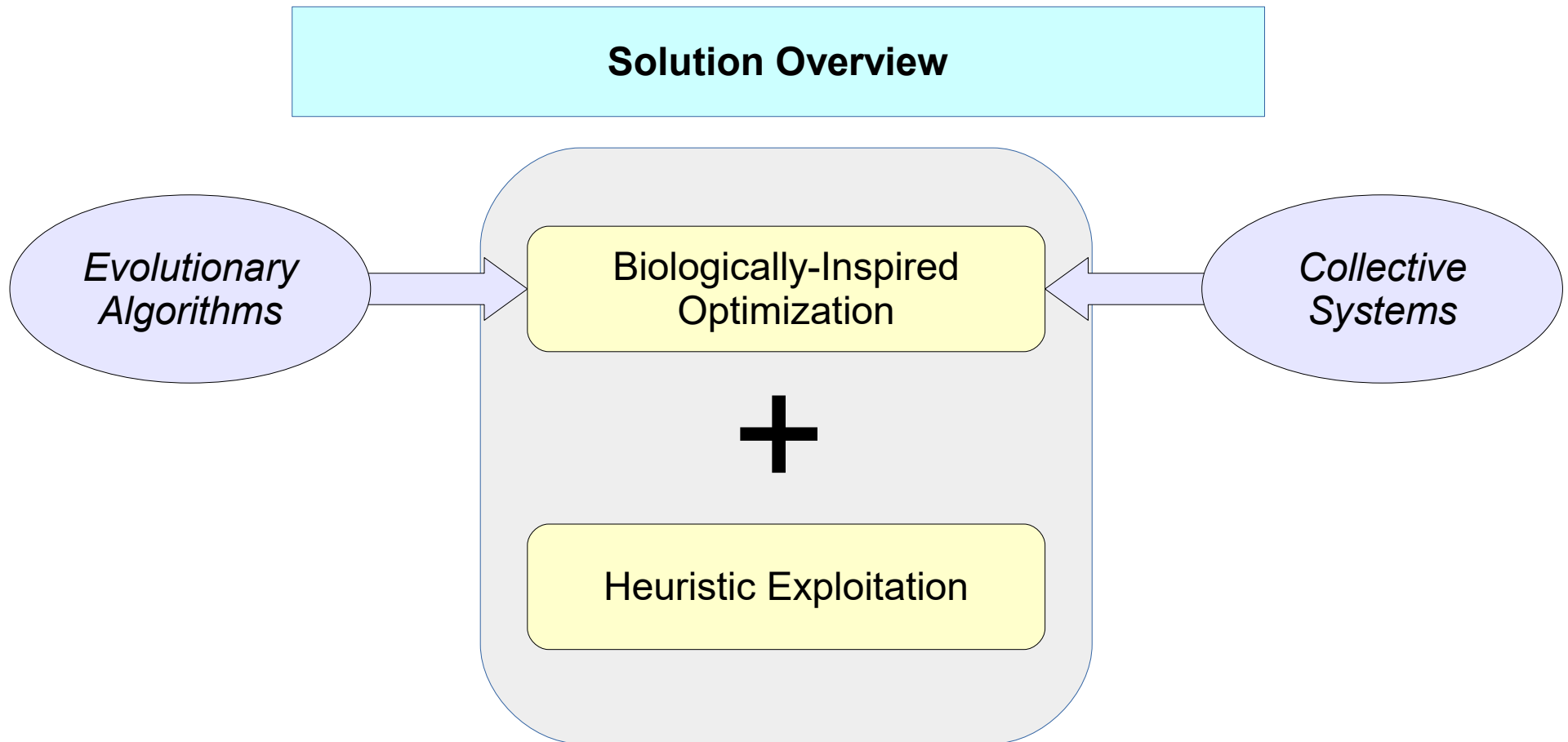
*The distance between consecutive solutions shall be  
as minimal as possible.*

Flexibility

*The algorithm maintains high robustness, scalability  
and convergency even for greatly varying  
kinematic structure.*



# Algorithmic Approach



# Algorithmic Approach

## Genetic Algorithms

- Developed by *J. F. Holland*
- Inspired by the theory of **natural evolution** as formulated by *C. Darwin*
- „*Survival of the fittest*“ and „*Diversity drives change*“
- Group of individuals within a population that evolves over many generations
- **Selection, Recombination and Mutation**

## Particle Swarm Optimization

- Developed by *J. Kennedy and R. Eberhart*
- Inspired by social emerging behaviour of **bird flocks** and **schools of fish**
- Rather simple organisms („particles“) collectively solve a complex problem
- **Velocity** and **direction** update according to success of **neighbouring** particles

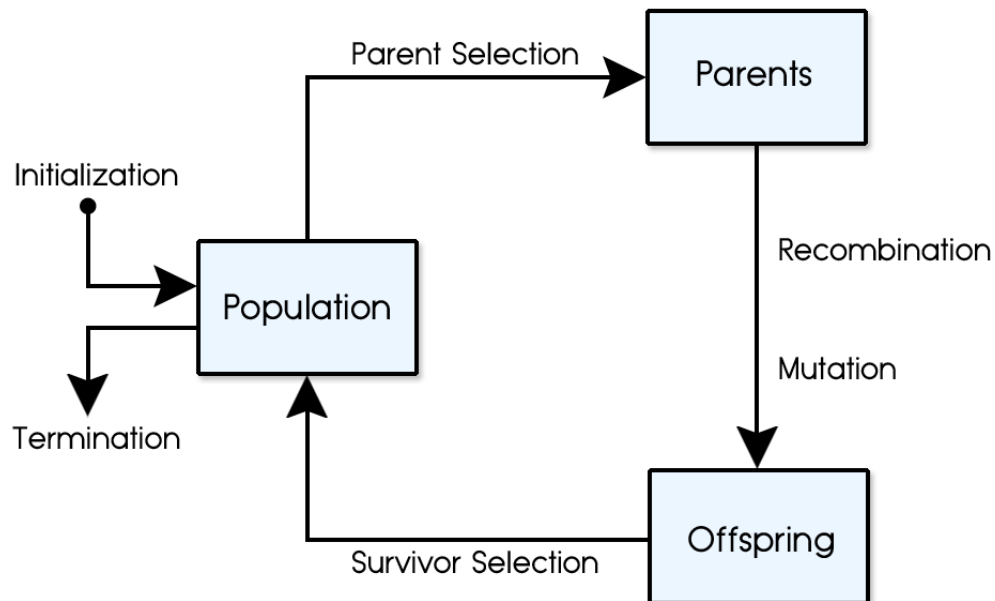
## Similarities

- Search space exploration by a **group of organisms**
- Solution quality determined by **fitness function**
- **Simultaneous search** for multiple local extrema
- High **robustness** and **scalability** as well as effectiveness for **multi-objective optimization**

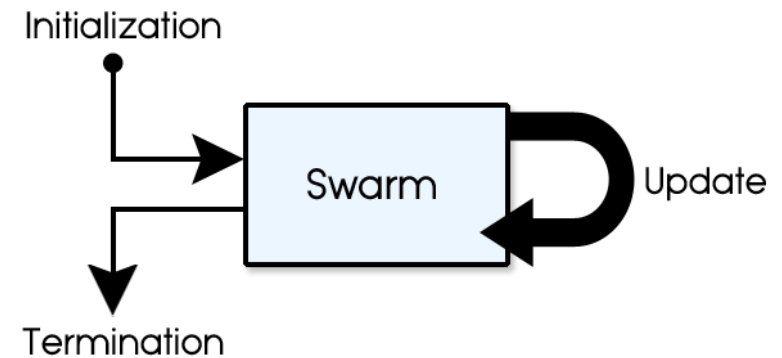
A.E. Eiben and J. E. Smith – *Introduction to Evolutionary Computing*, Springer, 2003  
D. Floreano and C. Mattiussi – *Bio-Inspired Artificial Intelligence*, MIT Press, 2008

# Algorithmic Approach

## Genetic Algorithms

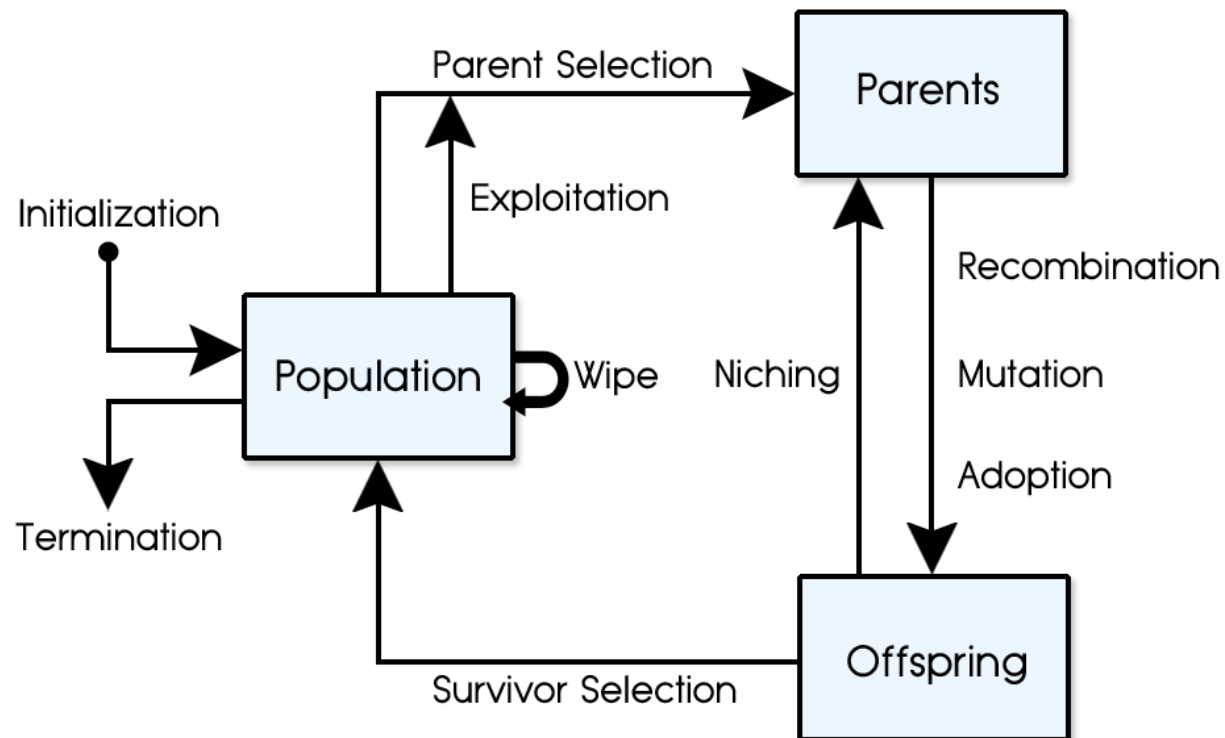


## Particle Swarm Optimization



# Algorithmic Approach

## Hybrid Genetic Swarm Algorithm



# Algorithmic Approach

## Encoding

Genotype  $x \rightarrow$  n-dimensional joint variable configuration

$$x = (x_1 \mid x_2 \mid x_3 \mid \dots \mid x_{n-1} \mid x_n)$$

- Independent of joint types (revolute, prismatic, ...)
- Joint limits directly incorporated (clipping)
- Allows algebraic vector calculations

# Algorithmic Approach

## Fitness Function

### Idea

Use randomized weight  $w$  for multi-objective optimization

→ Models **dynamic environment**

→ Determines individuals that are „**most responsive to change**“

→ Biologically plausible

Phenotype  $X$  → Cartesian configuration obtained by forward kinematics function  $f$

$$\mathcal{X} = f(x)$$

Fitness function  $\Omega$  measures fitness  $\pi$  under evolutionary target  $Y$

$$\pi = \Omega(\mathcal{X}) \quad \Omega : \begin{cases} d_t(\mathcal{X}, \mathcal{Y}) & \text{Position} \\ d_r(\mathcal{X}, \mathcal{Y}) & \text{Orientation} \\ wd_t(\mathcal{X}, \mathcal{Y}) + (1-w)d_r(\mathcal{X}, \mathcal{Y}) & \text{Pose} \end{cases}$$

# Algorithmic Approach

## Parent Selection

**Rank-based** parent selection from mating pool  $\Gamma$

$$\mathcal{S}_P : \mathcal{P}_{\{1,2\}} \leftarrow p(\Gamma_i) = \frac{\gamma - i + 1}{\sum_{i=1}^{\gamma} i}$$

- Independent of fitness value distribution
  - Sensitive to local extrema
  - Scales well with population size
  - No parameters required

# Algorithmic Approach

## Recombination

### Idea

Let offspring dive a little deeper into the direction that caused improvement within their parents

→ *Heuristic evolutionary gradients for continuous evolution dynamics*

→ **Evolutionary gradient** = amount of change during **mutation** and **adoption** (see later)

$$\mathcal{R} : x_i = \underbrace{w_i x_i^{\mathcal{P}_1} + (1 - w_i) x_i^{\mathcal{P}_2}}_{\text{Randomly interpolated genotype of parent chromosomes}} + \underbrace{r_{\mathcal{P}_1} g_i^{\mathcal{P}_1} + r_{\mathcal{P}_2} g_i^{\mathcal{P}_2}}_{\text{Randomly weighted swarm dynamics from evolutionary gradient } g} \quad \underbrace{\mathcal{R}_r = 1.0}_{\text{Constant recombination rate}}$$



# Algorithmic Approach

## Mutation

### Motivation

Fixed mutation rate and strength not suitable for arbitrary kinematic models

### Idea

→ Let the population itself determine the required amount of **exploitation** and **exploration** by an **extinction rate** that **controls mutation** independent from the problem dimensionality

$$\mathcal{M} : x_i \leftarrow x_i + \underbrace{w_i \mathcal{M}_\beta}_{\text{Mutation Strength}}$$

#### Mutation Strength

- Can grow arbitrarily small to exploit
- Exploration limited to the domain of the search space dimension

$$\mathcal{M}_\beta = \xi^{\mathcal{P}_\emptyset} \Delta n_i$$

$$\mathcal{M}_r = \underbrace{\mathcal{M}_\alpha \frac{1}{n}}_{\text{Mutation Rate}}$$

#### Mutation Rate

- Normalized between [1/n; 1.0]
- Adaptive to arbitrary dimensionality

$$\mathcal{M}_\alpha = \xi^{\mathcal{P}_\emptyset} (n - 1) + 1$$

$$\xi^{\Psi_i} = \underbrace{\frac{\pi_i + \pi_{\min} \left( \frac{i-1}{\psi-1} - 1 \right)}{\pi_{\max}}}_{\text{Extinction Factor}}$$

#### Extinction Factor

- Normalized between [0.0; 1.0]
- Measures relative quality of an individual within population
- Controls whole mutation

# Algorithmic Approach

## Adoption

### Idea

Let offspring **adopt characteristics** of parents and most successful performing prototypes

- Natural behaviour over lifetime
- Dynamic search space exploration
- Very similar to PSO

$$A : x_i \leftarrow x_i + \underbrace{w_i r_{\mathcal{P}} \left( \frac{x_i^{\mathcal{P}_1} + x_i^{\mathcal{P}_2}}{2} - x_i \right)}_{\text{Averaged gradient to parents}} + \underbrace{(1 - w_i) r_* (x_i^* - x_i)}_{\text{Gradient to best individual}}$$

} *Randomly interpolated*

# Algorithmic Approach

## Niching

### Pre-Selection

→ Immediately remove any parent whose fitness is worse than its offspring

### Goals

→ Encourages to keep track of multiple local extrema  
→ Avoid premature convergency

# Algorithmic Approach

## Survivor Selection

### Age-based with elitism

→ Merge all elites and newly generated offspring

### Goals

→ Let the fittest individuals (*elites*) survive in order not to lose the current evolutionary progress

# Algorithmic Approach

## Exploitation

### Motivation

Increase convergency by further improvement of already good solutions

### Approach

Iterative cyclic exploitation of **elite** individual genotypes

- 1. Randomly modify each gene by current fitness value into both domain directions*
- 2. Take modification that scored improvement*
- 3. Calculate averaged fitness value*

# Algorithmic Approach

## Initialization

### Biased population

*1 individual* → *currently assigned joint variable configuration*

*All others* → *randomly generated chromosomes*

## Termination

Determine whether desired accuracy in

**position** and **orientation**

is satisfied by the solution

## Wipe

### Problem

- Many local extrema within search space
- Escape from dead-end paths in good extrema might take too long
- Restart might be more efficient

### Approach

- Perform wipe (*restart*) of population if solution  
→ *could not be improved within last generation*  
**and**  
→ *can not be improved by exploitation*

# Algorithmic Approach

## Solution Filtering

### Motivation

Real-time interactive applications → best solution so far is required

### Problem

Randomized weights might replace elites by constant dynamics

### Solution


Determine joint variable solution by equally weighted objective function

- Remember best solution in history
- Let evolution continue approximation underneath

# Experiments and Results

## Environmental Setup

*Laptop – ASUS G751JY  
Intel Core i7-4720HQ (2.6 – 3.4 GHz)  
(Code running at single core implementation)*

 + URDF  
Importer + Kinematic  
Joint + IK  
Solver



# Experiments and Results

## Environmental Setup

**Kinematic Joint (Script)**

Type: Revolute

Connection: X 0 Y 0 Z 0

Axis Orientation: X 0 Y 0 Z 0

**X Motion**

State: Free

Max Velocity: 2

Max Acceleration: 2

Lower Limit: -0.25

Upper Limit: 1

Target Value: 0

**Y Motion**

State: Free

Max Velocity: 2

Max Acceleration: 2

Lower Limit: -0.5

Upper Limit: 1.5

Target Value: 0

**Z Motion**

State: Free

Max Velocity: 2

Max Acceleration: 2

Lower Limit: -1

Upper Limit: 0.75

Target Value: 0

**IK Solver (Script)**

Solver: [Empty]

Target: None (Transform)

Maximum Frame Time: 0.001

Objective: Pose

**Maximum Error**

Position: 0.001

Orientation: 0.01

**Algorithm**

Population Size: 12

Elites: 4

**Active Joints**

Torso (2 DoF)

Shoulder (2 DoF)

Elbow (1 DoF)

Degree of Freedom: 5

# Experiments and Results

## Environmental Setup



# Experiments and Results

*– Live Demos in Unity –*



- 1 - <https://www.youtube.com/watch?v=dRCY848mSLI>
- 2 - [https://www.youtube.com/watch?v=DZFeU\\_WZlhl](https://www.youtube.com/watch?v=DZFeU_WZlhl)
- 3 - <https://www.youtube.com/watch?v=8-kw7RsuD6A>
- 4 - <https://www.youtube.com/watch?v=OXtGbrl7qUQ>
- 5 - <https://www.youtube.com/watch?v=Gu0CBf18Zf0>
- 6 - <https://www.youtube.com/watch?v=a9QPXud-j0Q>

# Experiments and Results

## 1. Parameters

- *Balancing of population size and elites*
- *UR5 Arm*

## 2. Selective Study

- *All versus Nothing*
- *Selectively taking away improvements*
- *UR5 Arm*

## 3. Performance Study

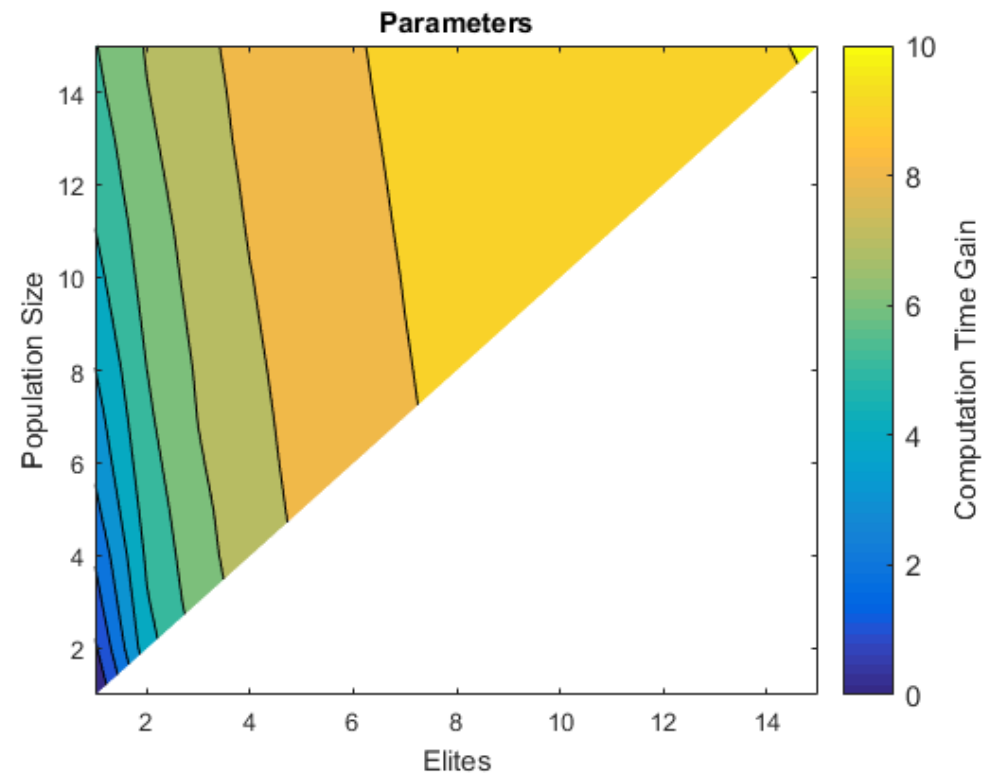
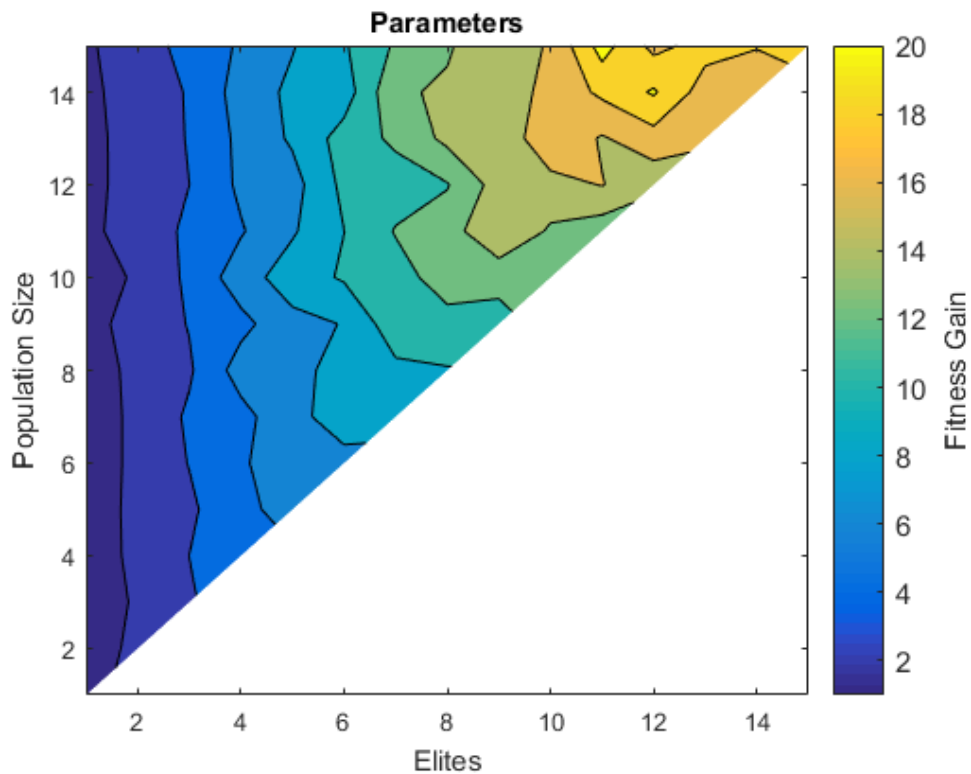
- *Success, Accuracy, Time, Displacement, Flexibility*
- *8 Models + 3 high dimensional chains*

## 4. Data Comparison

All samples generated as **random poses** or **continual trajectories** of joint space configurations

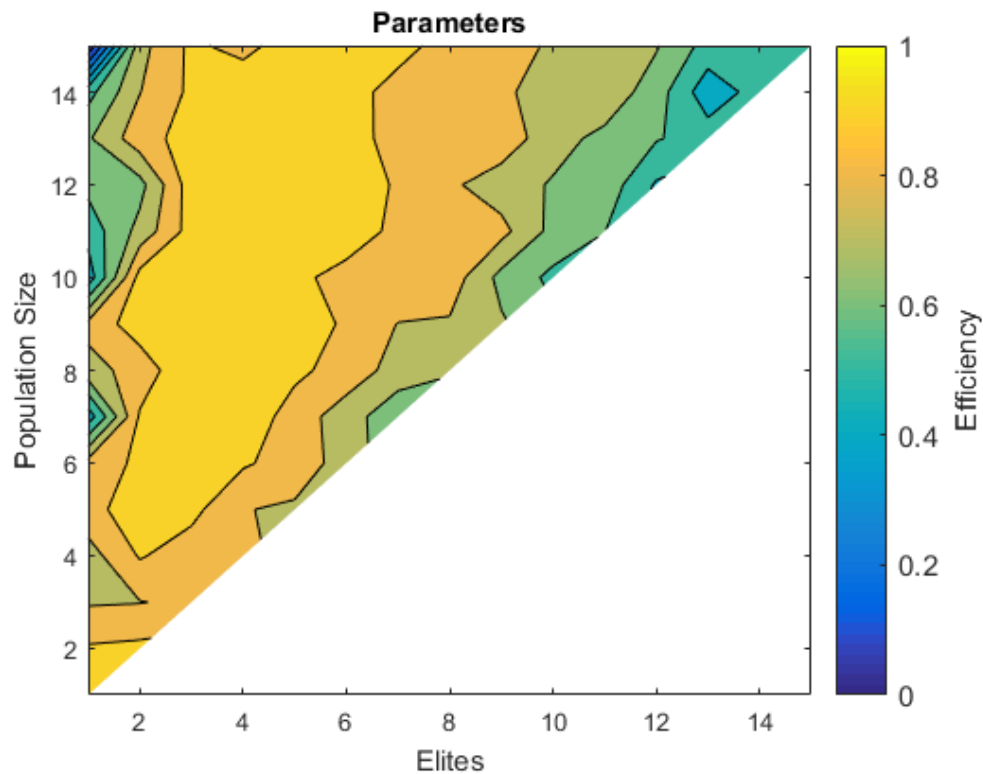
# Experiments and Results

## Parameters



# Experiments and Results

## Parameters



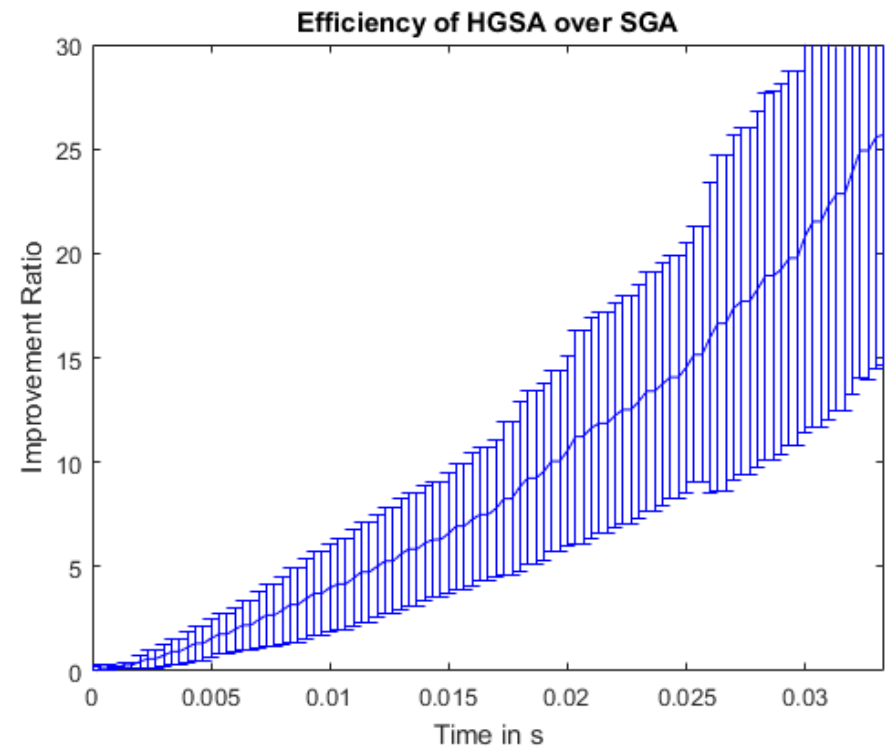
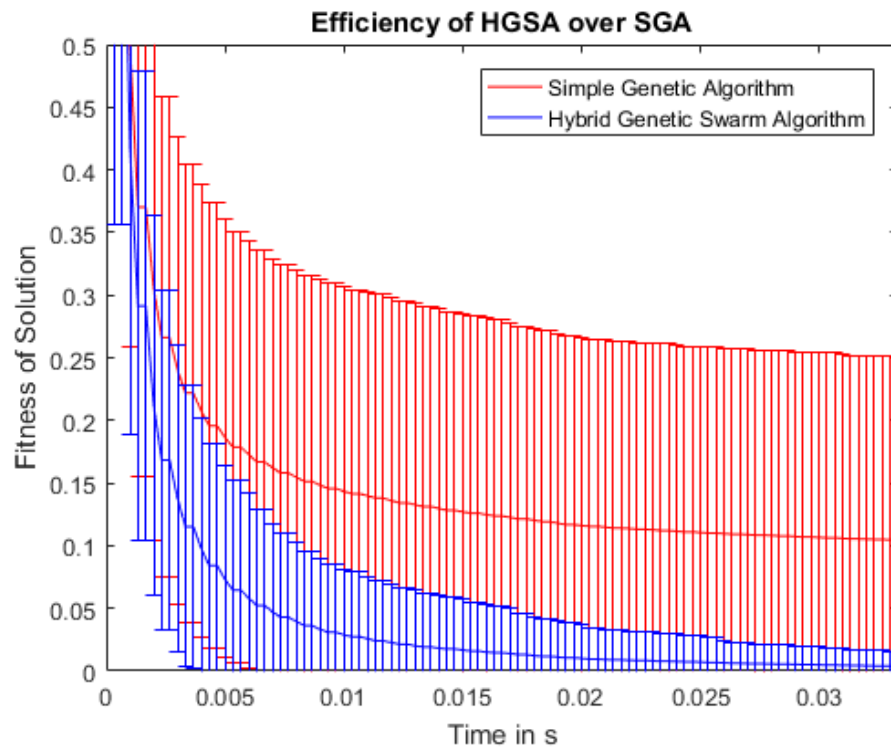
### Reasonable parameter choice

*Population Size* → 12

*Elites* → 4

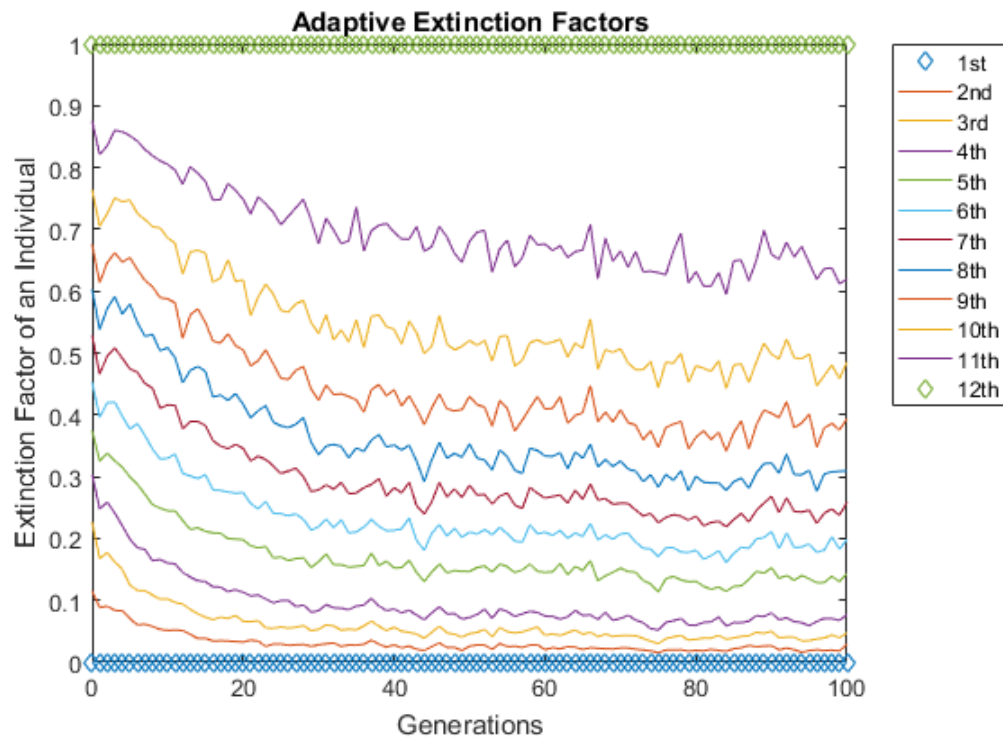
# Experiments and Results

## Selective Study – All versus Nothing



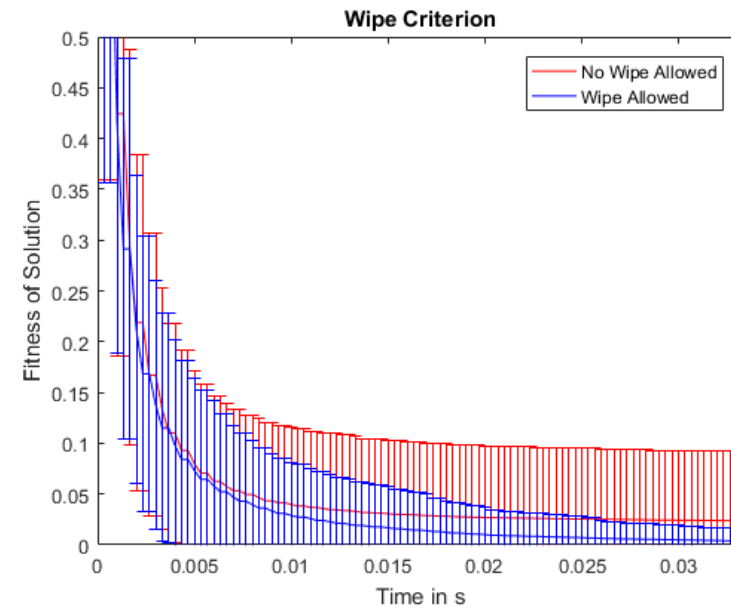
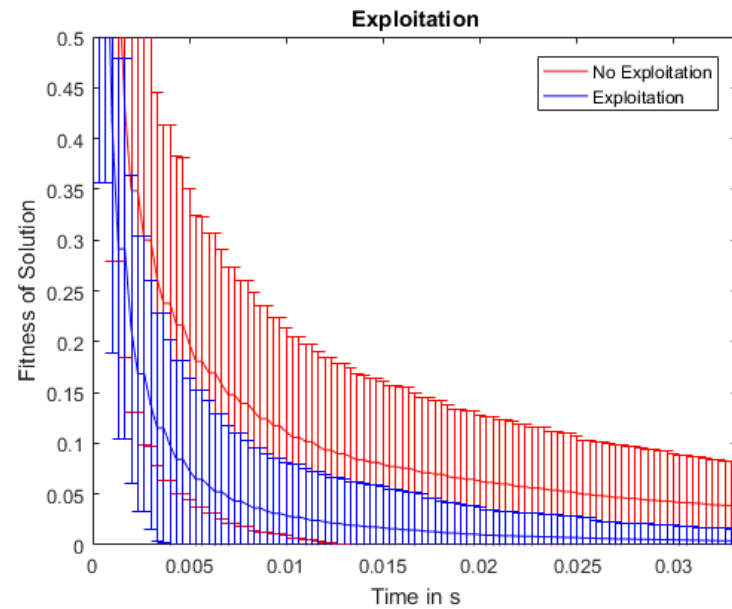
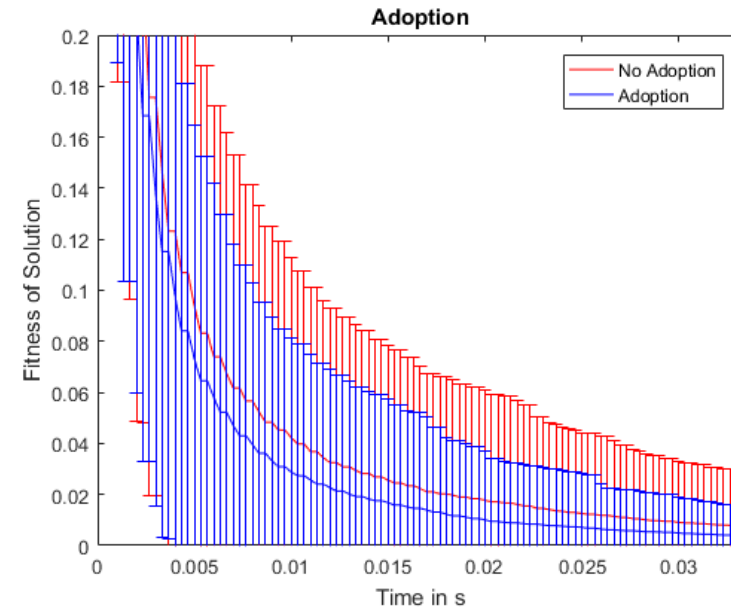
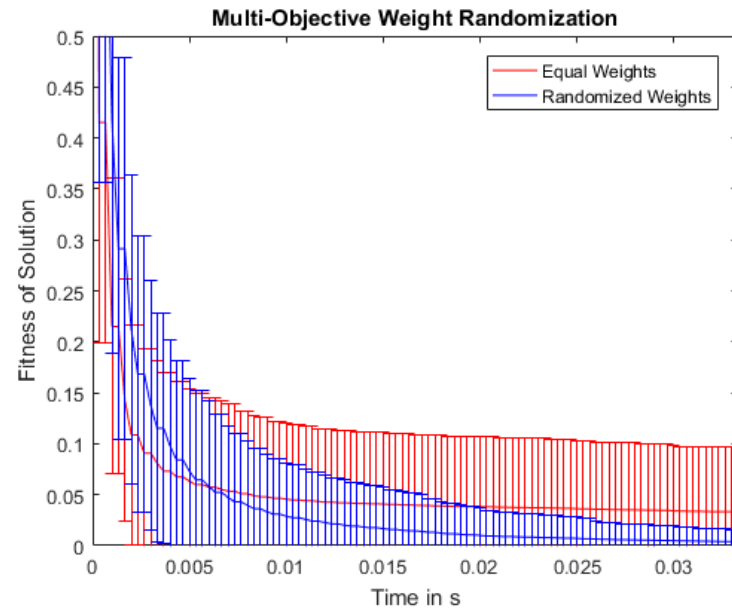
# Experiments and Results

## Selective Study – Extinction Factors



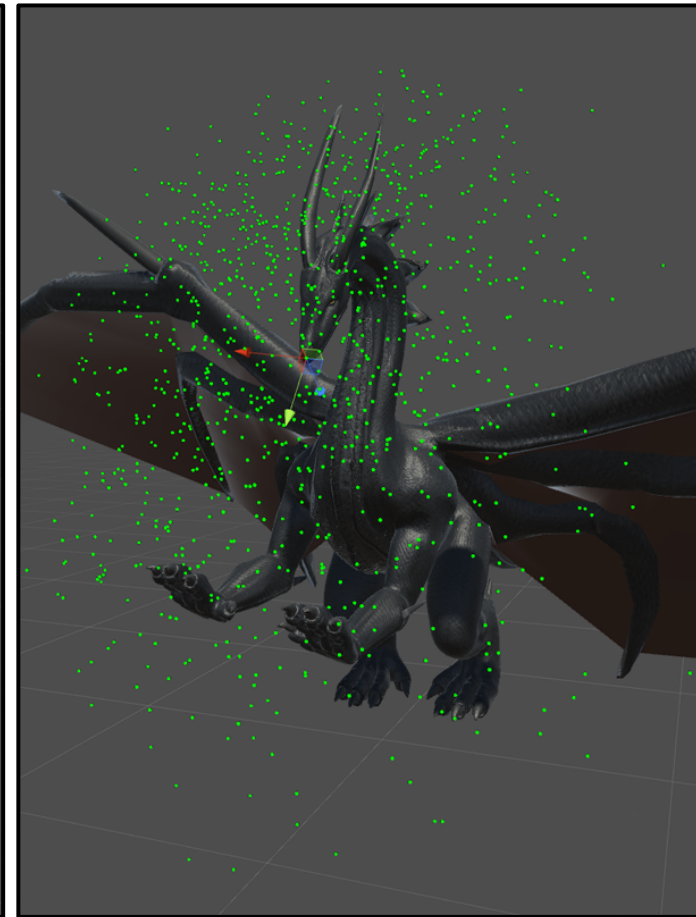
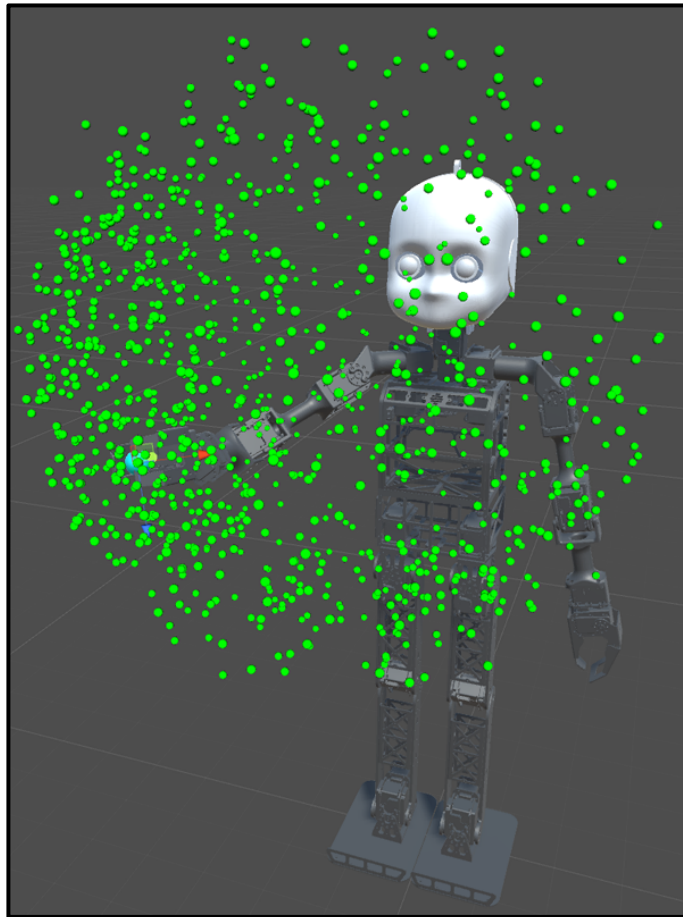
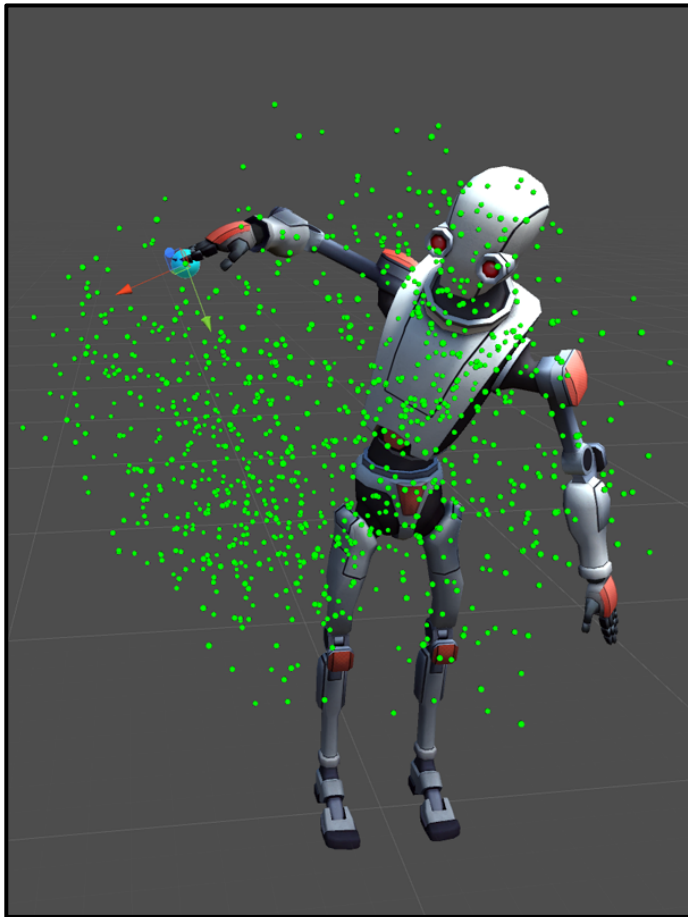
- Extinction factors **adapt** to the current evolutionary progress
- Maintains **explorative diversity**
- Ensures **local extrema sensitivity**





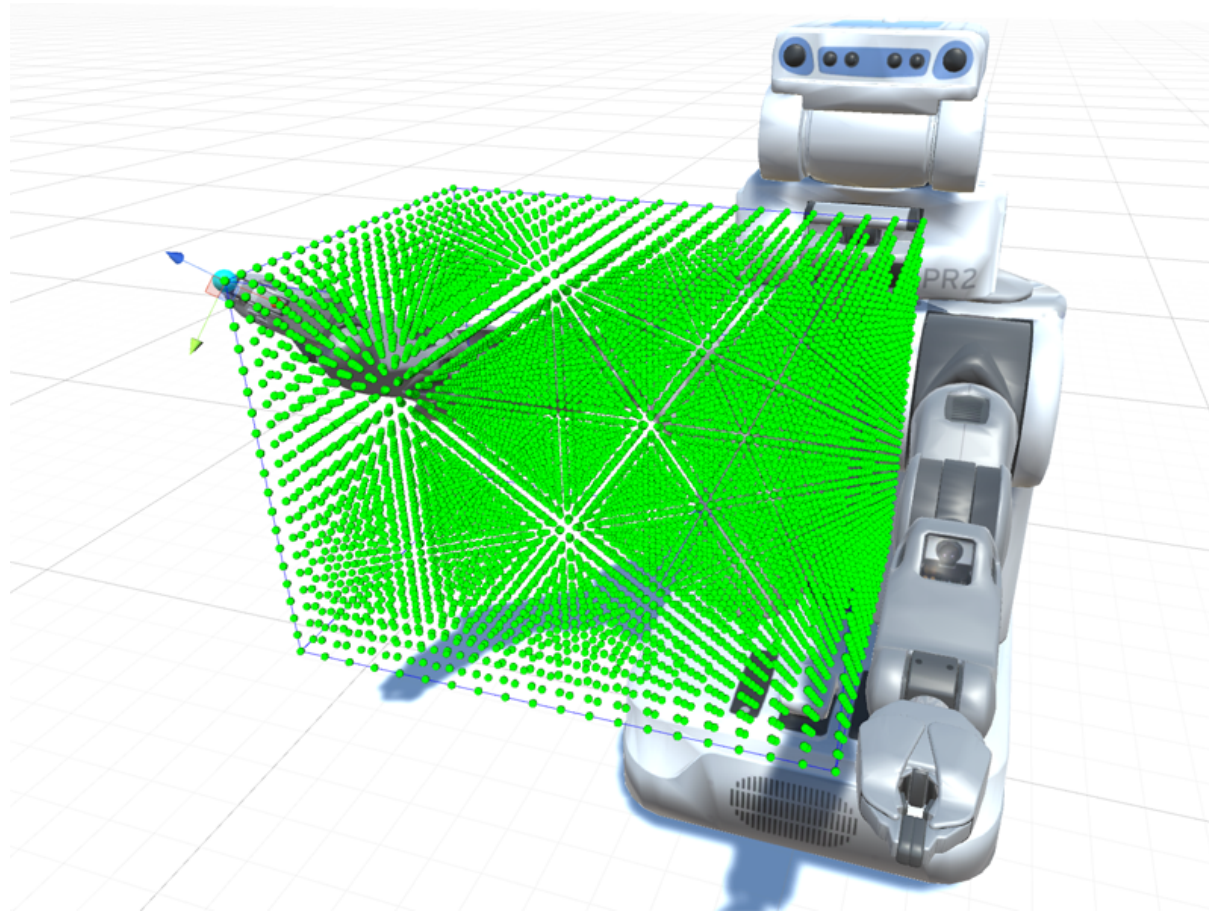
# Experiments and Results

## Performance Study – Success (*Pose Objective*)



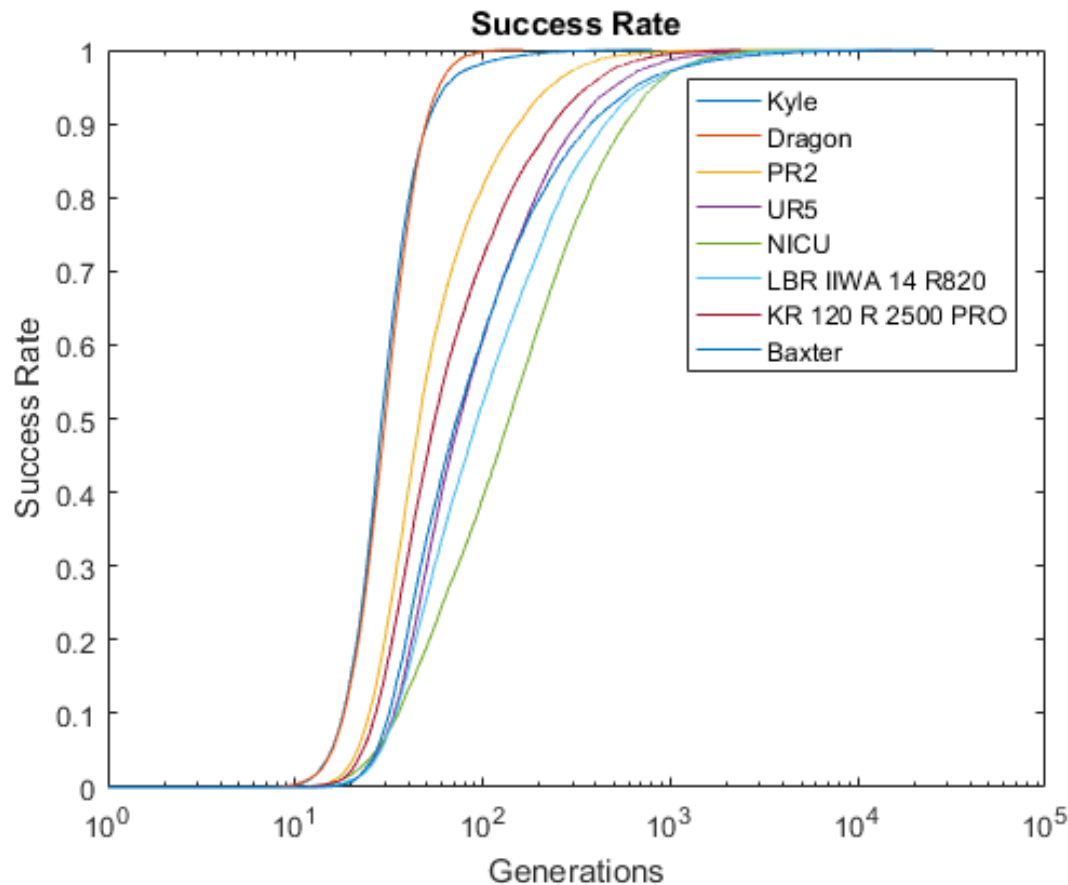
# Experiments and Results

## Performance Study – Success (*Position Objective*)



# Experiments and Results

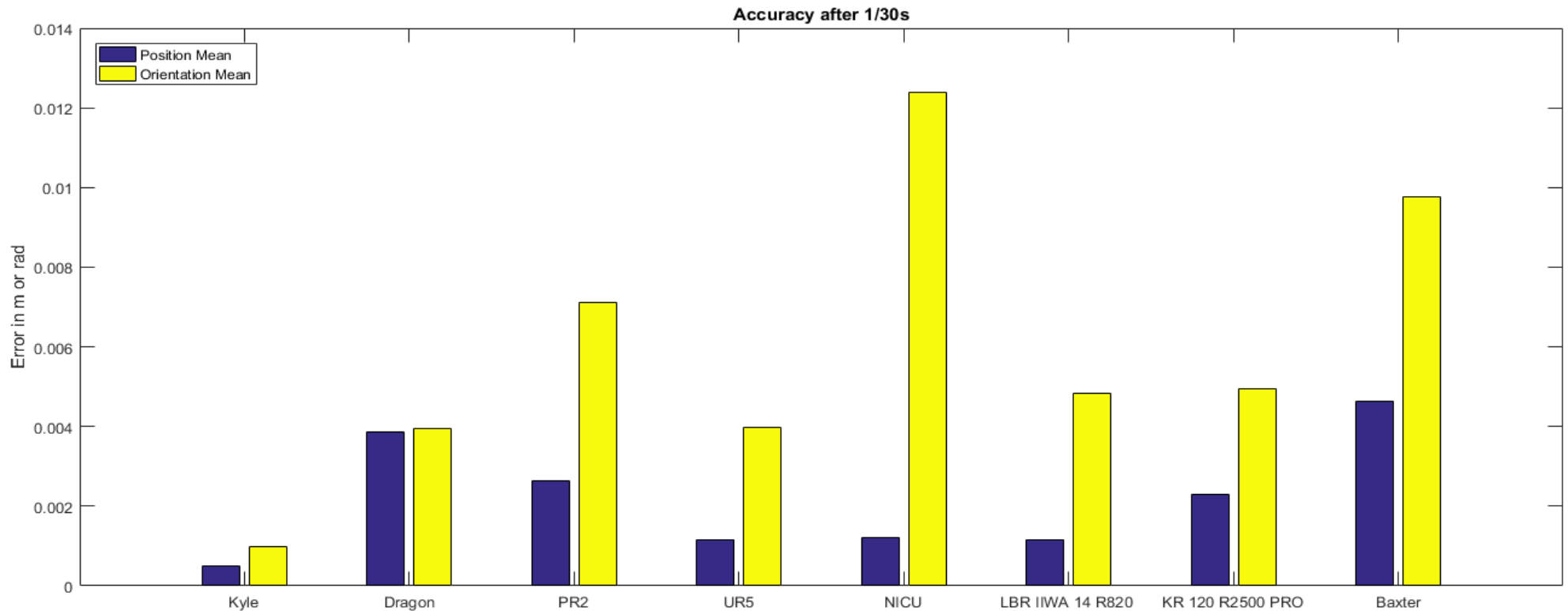
## Performance Study – Success



All 10.000 random poses for 8 different models were found.

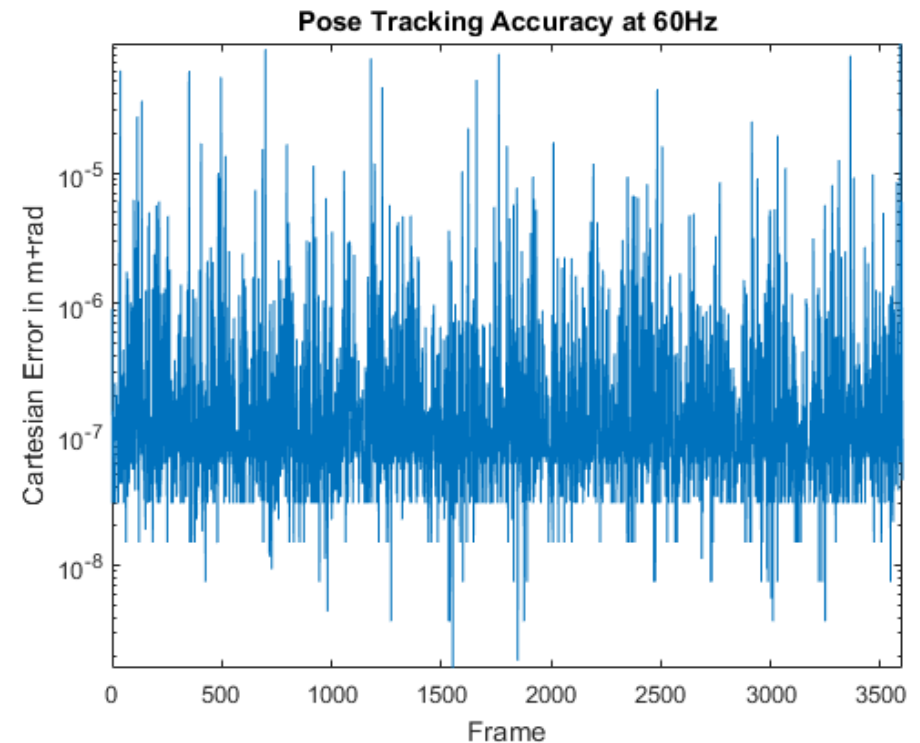
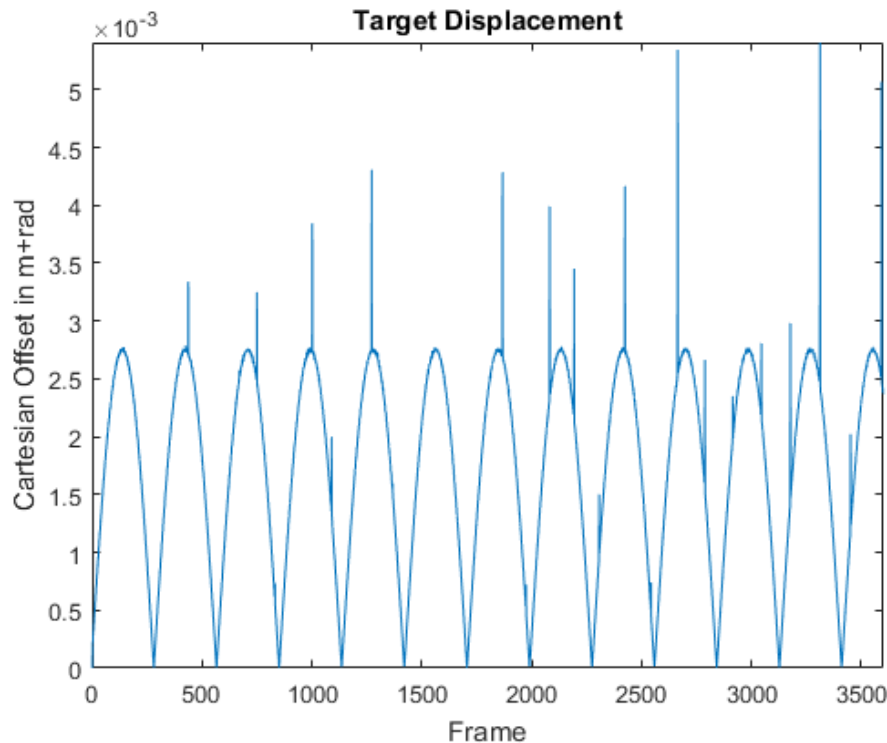
# Experiments and Results

## Performance Study – Accuracy



# Experiments and Results

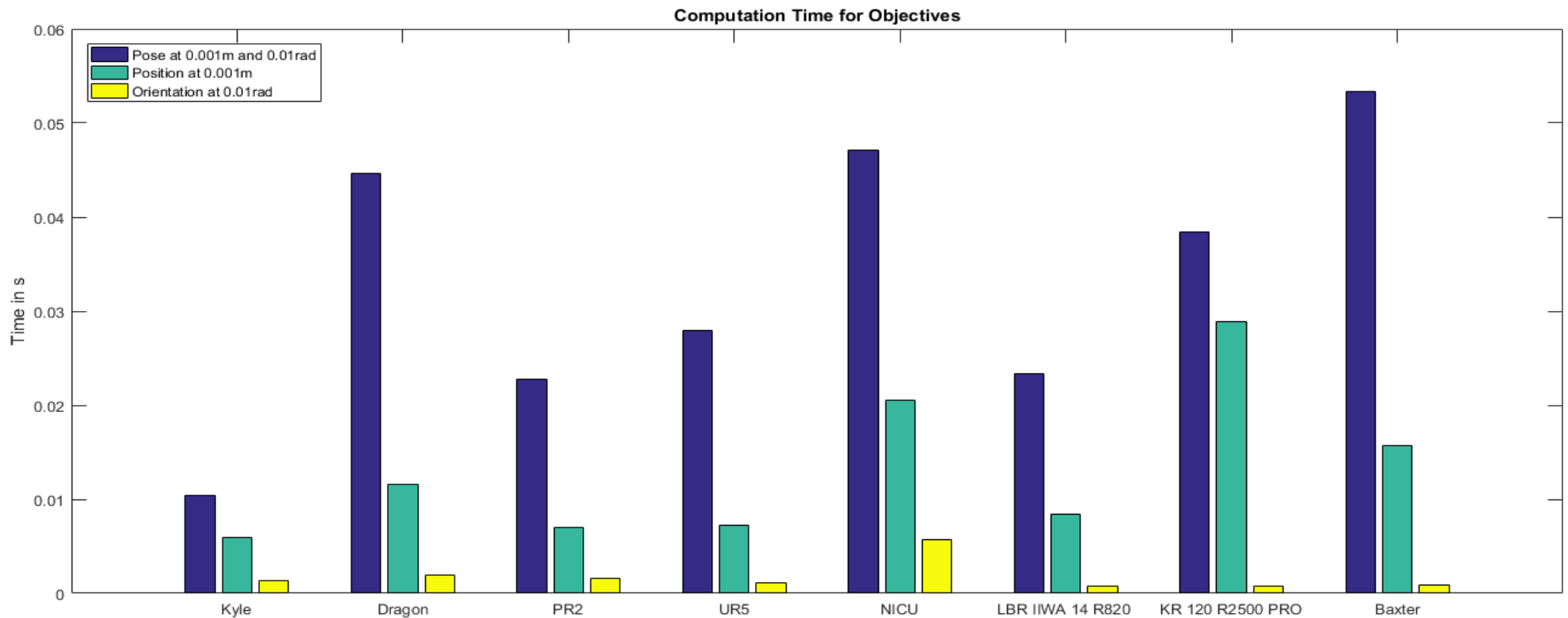
## Performance Study – Accuracy



→ Pose tracking with  $10^{-6}$  cartesian error (sum of position and orientation errors)

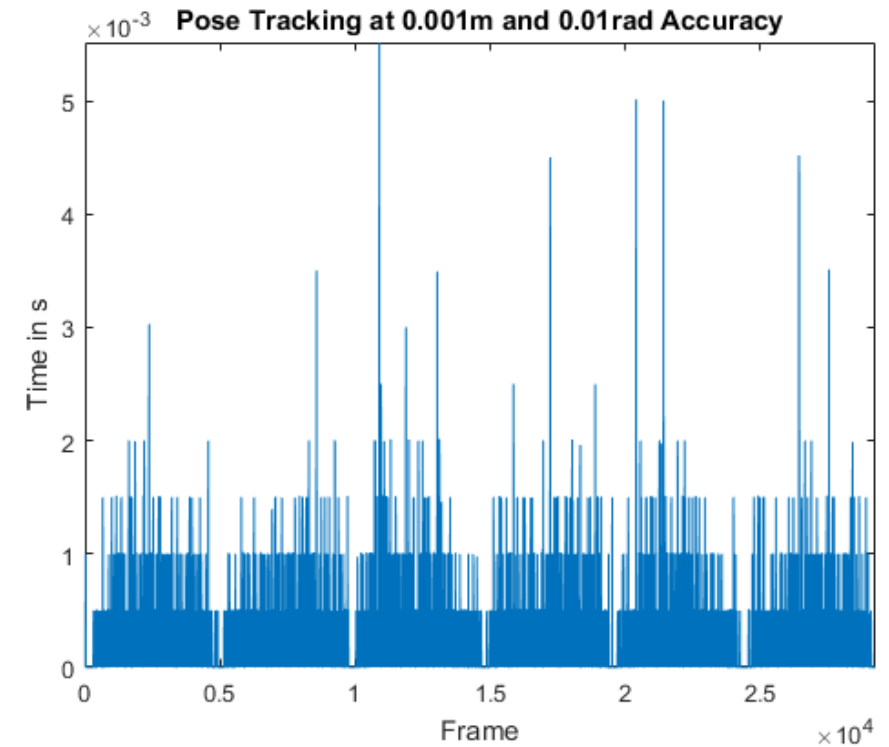
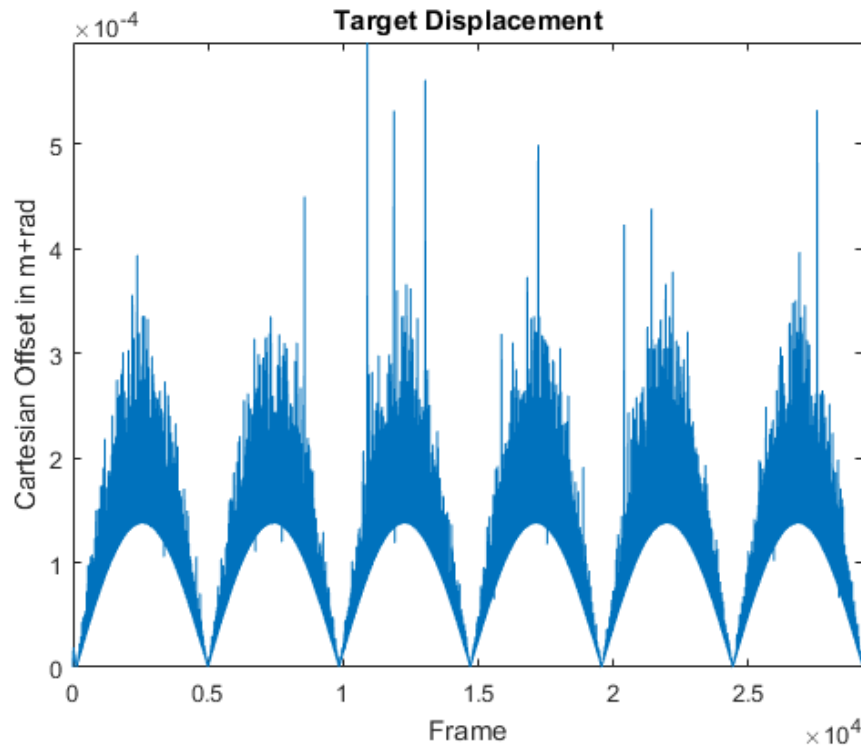
# Experiments and Results

## Performance Study – Time



# Experiments and Results

## Performance Study – Time

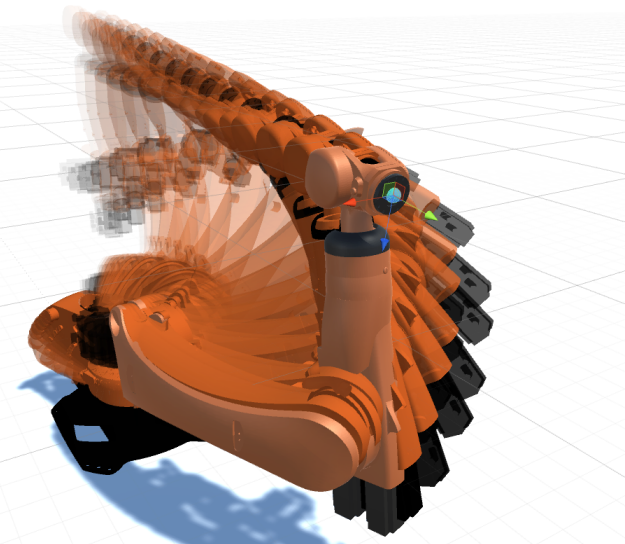
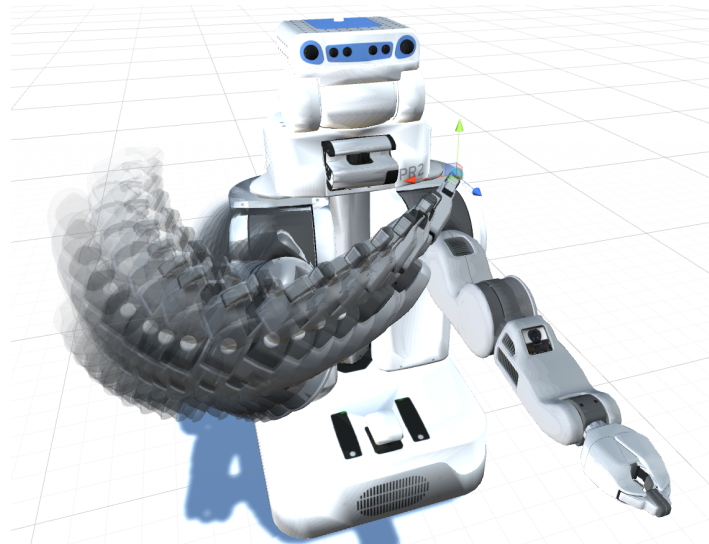
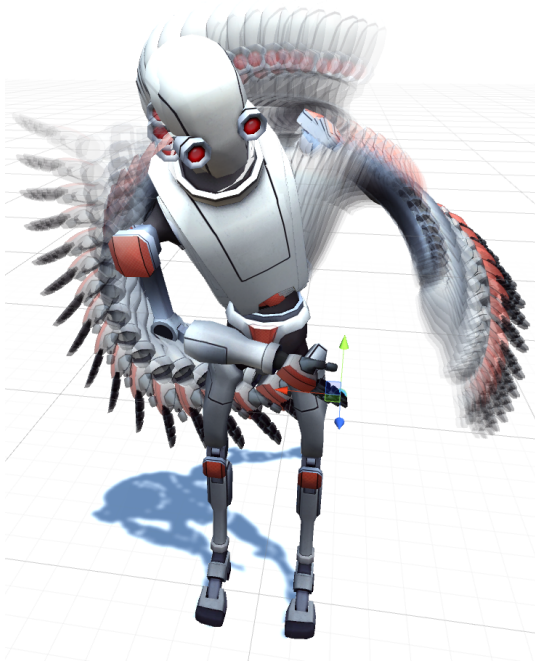


→ Accurate pose tracking at 1000-2000 Hz



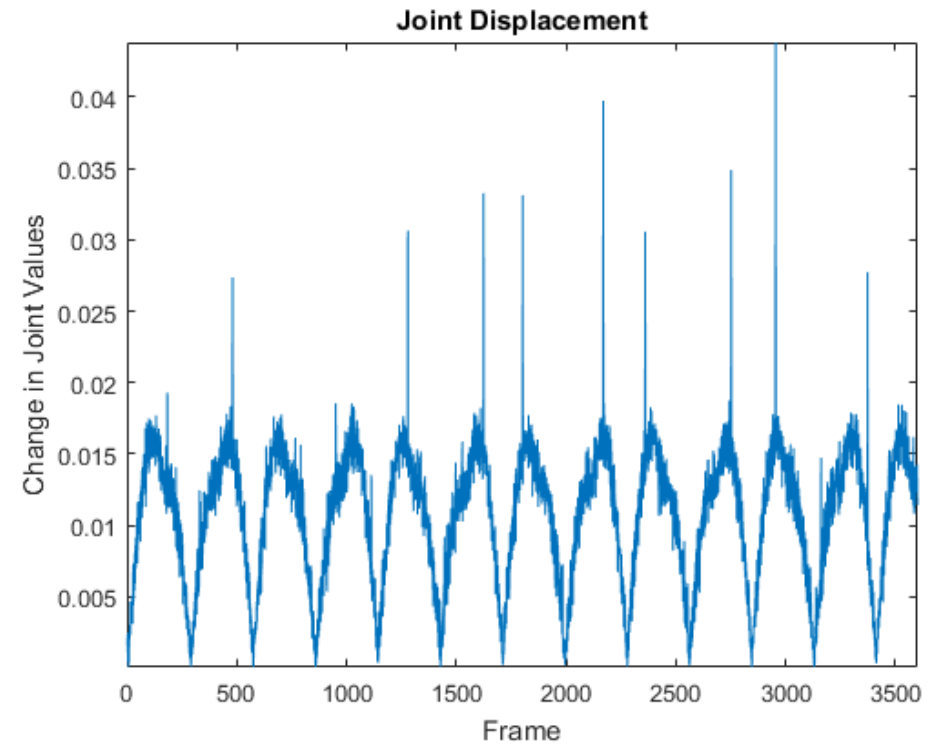
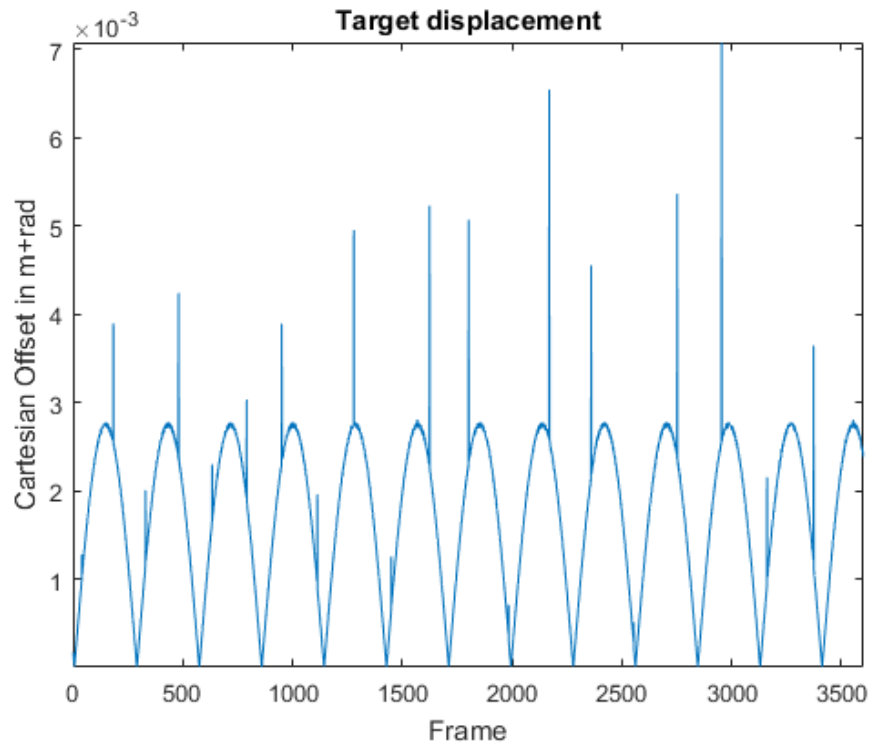
# Experiments and Results

## Performance Study – Displacement



# Experiments and Results

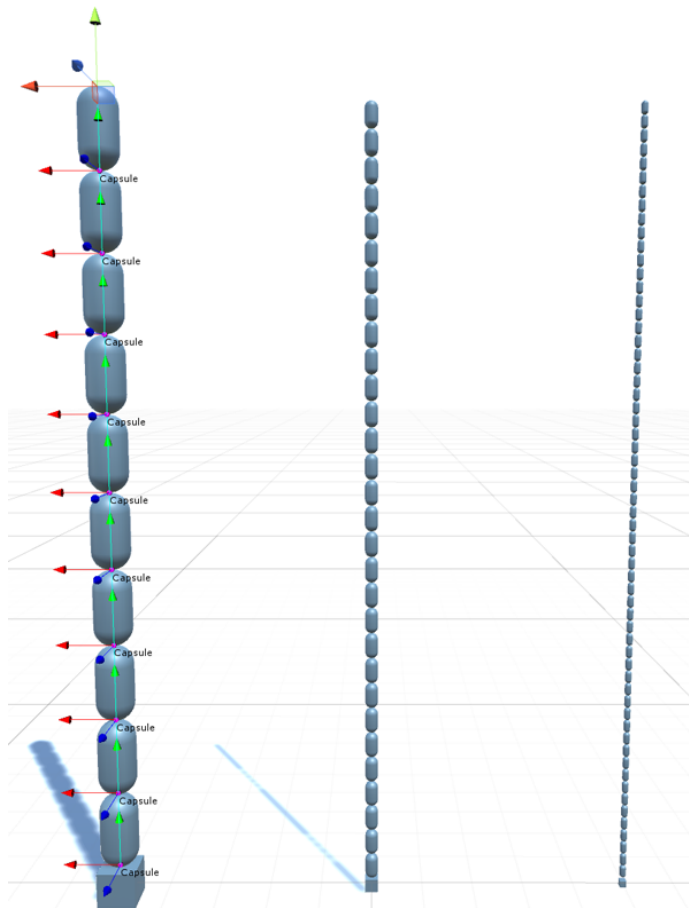
## Performance Study – Displacement



→ Responsive shape-resembling joint value change

# Experiments and Results

## Performance Study – Flexibility

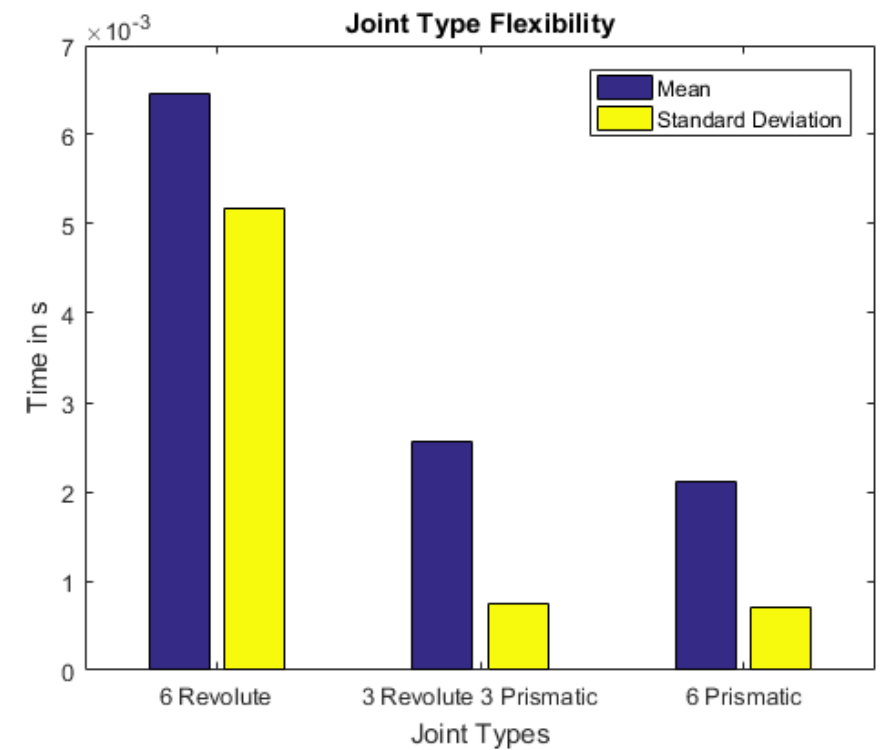
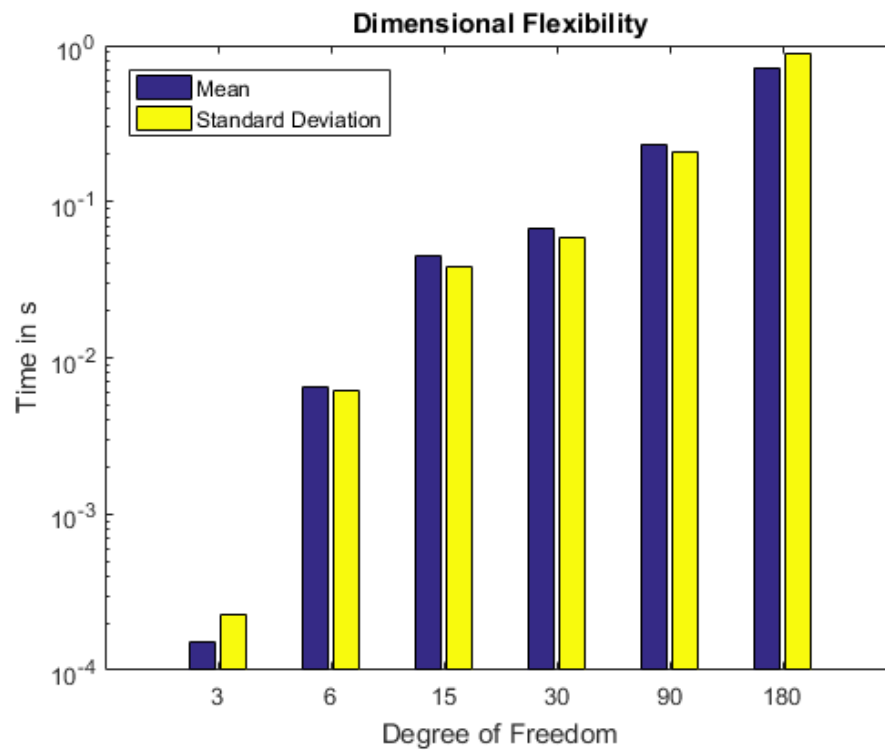


### Spherical Joints

left – 30 DoF  
middle – 90 DoF  
right – 180 DoF

# Experiments and Results

## Performance Study – Flexibility



# Experiments and Results


**Data Comparison**  
Reported efficiency in literature on various robot models

	Jacobian	CCD	FABRIK	GA	PSO	NN	HGSA
Accuracy ( <i>m</i>   <i>rad</i> )	0.00001	0.00001	0.0001	0.001	0.001	0.01	0.001
Time ( <i>ms</i> )	<1 - 10	1 - 100	10-50	50 - 500	30 - 600	-	10 - 60
Robustness	Medium	Medium	Low	High	Medium	Low	High
Scalability	--	-	+	+	++	--	++

# Experiments and Results

## Data Comparison Orocos' KDL vs TRAC-IK vs HGSA

	Orocos' KDL <i>(no joint limits)</i>		TRAC-IK <i>(Beeson &amp; Ames, 2015)</i>		HGSA	
	Success	Time	Success	Time	Success	Time
PR2	83.14%	1.37ms	99.84%	0.59ms	100%	22.75ms
LBR IIWA	37.71%	3.37ms	99.63%	0.56ms	100%	23.39ms
UR5	35.88%	3.30ms	99.55%	0.42ms	100%	27.88ms
Baxter	61.07%	2.21ms	99.17%	0.60ms	100%	53.33ms

  
**Scalability?**  
 Joint displacement due to  
 random restarts?

# Experiments and Results

## Data Comparison PASO vs HGSA

	PASO <i>(Collins &amp; Shen, 2016)</i>	HGSA
	Time	Time
30 DoF	1.57s	0.066s
90 DoF	7.46s	0.233s
180 DoF	37.03s	0.717s

# Conclusion

## General

- *Universal IK solver for kinematic chains*
- *Arbitrary joint types, link geometry and degree of freedom*
- *Algorithm based on biologically-inspired evolutionary and collective concepts*

## Algorithmic Improvements

Extinction Factors

Swarm Adoption

Heuristic Exploitation

Multi-Objective Weight Randomization

Wipe Criterion

## Results

- *100% success rate at high accuracy and real-time capability*
- *Minimal joint displacement and high scalability*
- *Maintains performance under various kinematic models*
- *Can compete with the State-of-the-Art*



# Future Work

## Extinction Factor Study

→ Analyse loss by adaptive mutation control in contrast to model-specific optimized parameters

## Parallel GPU / Multi-Core Implementation

- >95% of time is required by fitness calculation (*FK equations*)
  - Reimplement the algorithm in C++/Python
- Efficient parallel instead of sequential single-core fitness calculation

## Multiple End Effectors

- Currently, only serial kinematic chains are supported
- Extend algorithm to solve for multiple end effectors simultaneously

## Path Planning

- Extend algorithm for path-planning tasks

## Collision Avoidance

- Incorporate link geometry to filter solutions that would result in collisions

## Neural Learning

- Learn mappings from previously evolved joint configurations and trajectories

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