Nonparametric Filter

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Outline

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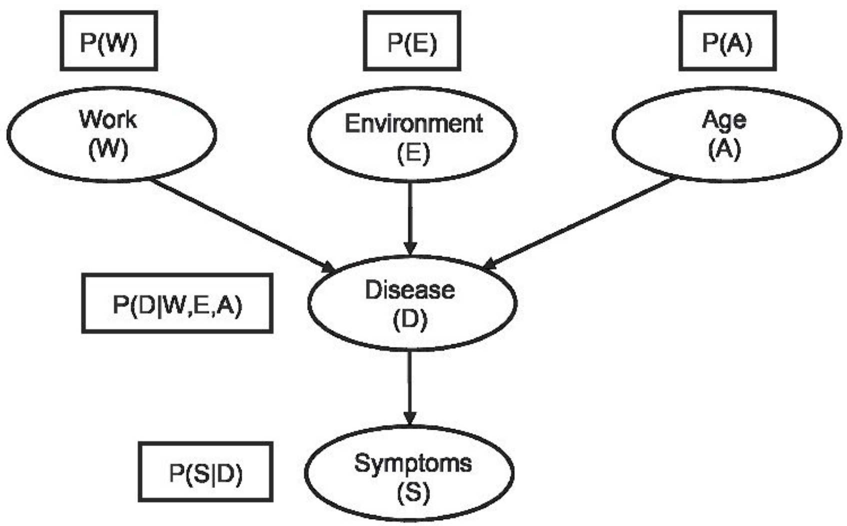
Bayesian Network

- **Graphical model of conditional probabilistic relation**
- **Directed acyclic graph (DAG)**

$$G = (V, E)$$

V: set of random variables

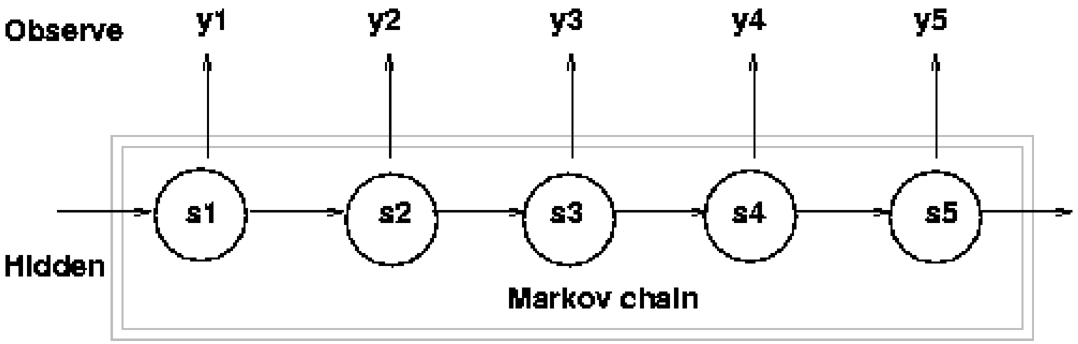
E: set of conditional dependencies



http://www.intechopen.com/books/current-topics-in-public-health/from-creativity-to-artificial-neural-networks-problem-solving-methodologies-in-hospitals

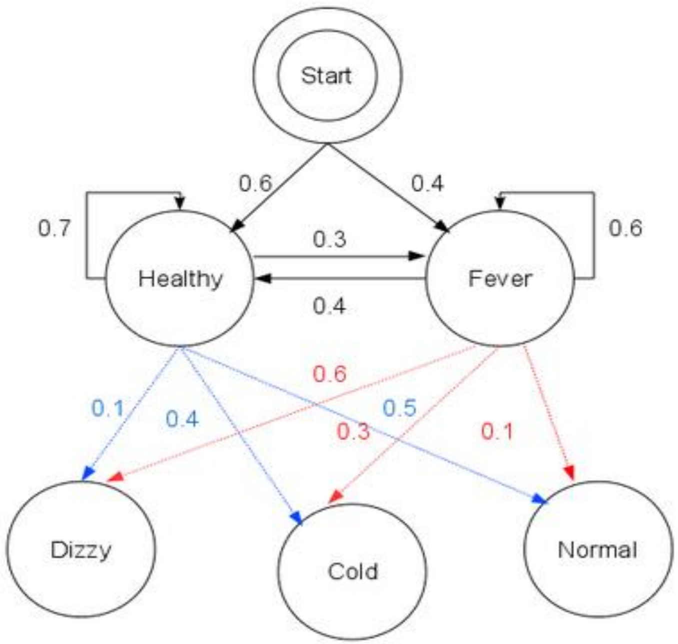
Hidden Markov Model

- Particular kind of Bayesian Network
- Modelling time series data



http://sites.stat.psu.edu/~jiali/hmm.html

Hidden Markov Model



https://en.wikipedia.org/wiki/Viterbi_algorithm#Example

Hidden Markov Model

Observing a patient for 3 days:

+ Day 1: Cold

+ Day 2: Normal

+ Day 3: Dizzy

Question:

1) Most likely sequence of health condition of the patient in last 3 days ?
2) Most likely health condition of the patient in the 4^{th} day ?

State space

- Quantities that cannot be directly observed but can be inferred from sensor data
- Examples: position and direction of robot in a room
- Notation:

$$X = \{x_1, x_2, \dots x_t\}$$

$$P(X = x_t): probabilty\ of\ sate\ equals\ to\ x\ at\ time\ t$$

Measurement (Observation)

- Environment data provided by robot sensor
- Examples: distance to ground, camera images
- Notation:

$Z = \{z_1, z_2, \dots, z_t\}$

$P(Z = z_t): probabilty\ of\ measurement\ equals\ to\ z\ at\ time\ t$

Control data

- Information about the change of state in the environment
- Examples: velocity of robot, temperature of a room, an action of robot on environment objects
- Notation:

$$U = \{u_1, u_2, \dots, u_t\}$$

$$P(U = u_t): probabilty\ of\ measurement\ equals\ to\ z\ at\ time\ t$$

Probabilistic Generative Laws

- State can be constructed on all past states, measurements and controls:

$$P(X = x_t) = P(X = x_t | x_{0:t-1}, z_{0:t-1}, u_{0:t-1})$$

- Markov assumption:

$$P(X = x_t) = P(X = x_t | x_{t-1}, u_t)$$
$$P(Z = z_t) = P(Z = z_t | x_t)$$

Belief distribution

- Belief:

- Internal knowledge of the robot about the true state
- Represent probability to each possible true sate
- Notation:

$$bel(x_t) = p(x_t | z_{1:t}, u_{1:t})$$

- Prediction:

$$\overline{bel}(x_t) = p(x_t | z_{1:t-1}, u_{1:t})$$

- Correction:

$$bel(x_t) = \mathrm{F}(\overline{bel}(x_t))$$

Bayes Filter algorithm (continuous case)

1: $Func\_continous\_Bayes\_filter\ (bel(x_{t-1}), u_t, z_t)$

2: $\quad for\ all\ x_t\ do$

3: $\qquad \overline{bel}(x_t) = \int p(\ x_t | u_t, x_{t-1}) bel(x_{t-1}) dx$

4: $\qquad bel(x_t) = normalizer\ * p(z_t | x_t)\ \overline{bel}(x_t)$

5: $\quad end$

6: $\quad return\ bel(x_t)$

Bayes Filters algorithm (discrete case)

$Func\_discrete\_Bayes\_filter(p_{k,t-1}, u_t, z_t)$

$\quad for\ all\ k\ do$

$\quad\quad \overline{p_{k,t}} = \sum p(\,x_t\,|\,u_t, X_{t-1} = x_i)p_{i,t-1}$

$\quad\quad p_{k,t} = normalizer\ * p(z_t\,|\,x_t)\overline{p_{k,t}}$

$\quad end$

$\quad return\,\overline{p_{k,t}}$

Histogram Filter

- Discrete Bayes filter estimation for *continuous* state spaces

- State space decomposition:

- $Range(X_t) = \{x_{1,t} \cup x_{2,t} \cup \dots x_{M,t}\}$
- $For\ every\ i\ \neq k: x_{i,t} \cap x_{k,t} = \emptyset$

- In each region the posterior is a piecewise constant density:
- $For\ every\ state\ x_t\ belongs\ to\ k^{th}\ region$:

$$p(x_t) = \frac{p_{k,t}}{|x_{k\ t}|}$$

Histogram filter

- **Problem:** prior information is defined for individual states, not for region !

- Refer to line 3, 4 of discrete Bayes filter algorithm

- **Solution:** approximating density of a region by a *representative* state of that region.

$$\widehat{x_{k,t}} = \frac{\int_{x_{k,t}} x_t \, dx_t}{|x_{k,t}|}$$

Histogram filter

- Approximation of density values for regions:

$$p(z_t|x_{k,t}) \approx p(z_t|\widehat{x_{k,t}})$$

$$p(x_{k,t}|u_t, x_{i,t-1}) \approx normalizer \ * \ p(\widehat{x_{k,t}}|u_t, \widehat{x_{i,t-1}})$$

- Precondition: all regions must have the same size.

- Now discrete Bayes filter algorithm is applicable !

Binary Bayes filter with Static State

- Belief is a function of measurement:

$$bel_t(x) = p(x|z_{1:t}, u_{1:t}) = p(x|z_{1:t})$$

- **General algorithm:**

1: $Func\_binary\_Bayes\_filter(l_{t-1}, z_t)$

2: $$l_t = l_{t-1} + \log\frac{p(x|z_t)}{1 - p(x|z_t)} - \log\frac{p(x)}{1-p(x)}$$

3: $return\ l_t$

- **Log odds ratio**

$$l(x) = \log \frac{p(x)}{1 - p(x)}$$

- Avoids truncation problems when probabilities close to 0 or 1

- **Inverse measurement model:**

- Reduce complexity by using probability of state given measurement data

- Example: infer state of a door in an image is much easier than infer an image from all other images of a close/open door.

Example of Binary filter: Occupancy grid mapping

- **Estimate (generate) map from (noisy) sensor measurement data and robot position**
- **General algorithm:**

$$\boldsymbol{p(Map = M|z_{1:t}, x_{1:t}) = \prod_{c} p(Cell = c\ is\ occupied|z_{1:t}, x_{1:t})}$$

$\boldsymbol{p(Cell = c\ is\ occupied|z_{1:t}, x_{1:t})}$ **is a binary estimation problem**

Particle filter algorithm

- **Represent the posterior density by a set of weighted random particles**
- **General algorithm:**

$Func\_Particle\_filter(X_{t-1}, u_t, z_t)$

$\quad \overline{X_t} = X_t = \emptyset$

$\quad for\ i = 1\ to\ M\ do$

$\qquad sample\ x_t^i \sim p(x_t | x_{t-1}^i)$

$\qquad w_t^i = p(z_t\ | x_t^i)$

$\qquad \overline{X_t} = X_t + (x_t^i, w_t^i)$

$\quad endfor$

$\quad for\ i = 1\ to\ M\ do$

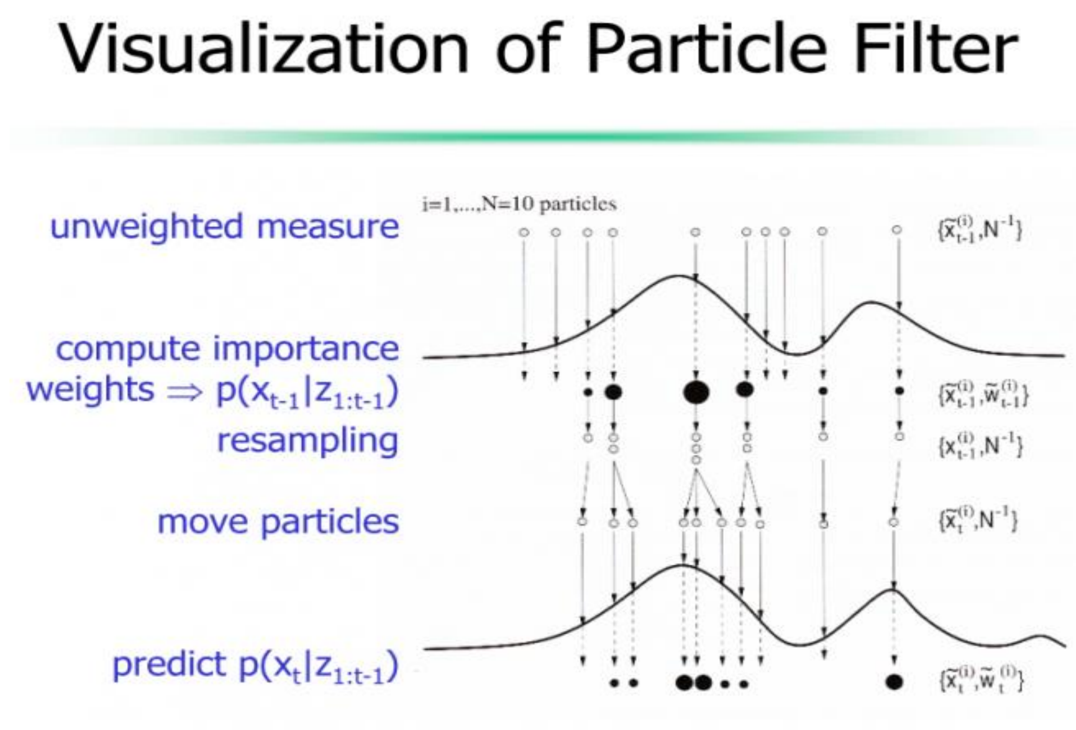
$\qquad draw\ i\ with\ probability\ \propto w_t^i$

$\qquad add\ x_t^i\ to\ X_t$

$\quad endfor$

$\quad return\ \ X_t$

Particle filter algorithm



http://www.juergenwiki.de/work/wiki/doku.php?id=public:particle_filter

Properties of Particle filter algorithm

- **Degree of freedom:**

- Because of normalization we lost one degree of freedom:

$$\deg = M \ - 1$$

- **Identical particles after resampling phase**:

- Resampling with probability proportional to weight: after every iteration we failed to draw one or more state sample

Properties of Particle filter algorithm

- **Deterministic sensor:**

- Sensor with noise-free range: measurement data is zero for most of state !

$\Rightarrow$All weights become zero.

- **Particle deprivation problem:**

- Resampling can wipe out all particles near the true state

$\Rightarrow$incorrect states have larger weight !

Application of Particle filter

- **Tracking the state of a dynamic system modeled by a Bayesian Network:** Robot localization, SLAM, robot fault diagnosis.

- **Image segmentation:** by generating a large number of particles and gradually focus on particle with desired properties

$\Rightarrow$**Image processing, Medial image analysis**

Summary

- Nonparametric filters represent posterior state as a function of previous poster state
- Nonparametric filters does not rely on a fixed functional form of posterior
- Histogram filter and Particle filter represent state space and posterior as a finite set of data
- There is usually a trade-off between efficiency and level of detail of data

References

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