Surface Reconstruction with Alpha Shapes

Erik Fließwasser

University of Hamburg
Faculty of Mathematics, Informatics and Natural Sciences
Department of Informatics
Technical Aspects of Multimodal Systems

07. December 2015

Outline

Motivation

How to reconstruct a surface from a given set of points?

INPUT

range or contour data
(e.g. from laser range finder)

$\Longrightarrow$

OUTPUT

(most optimal) approximation
of the real surface

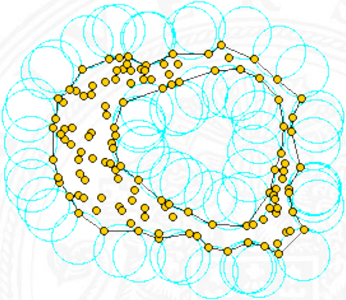
Point set [4]

Alpha Shape [4]

Motivation

The ice cream analogy

- ice cream with solid chocolate chips
- spherical ice spoon
- curve out all parts of the ice cream without touching the chocolate chips
- straighten all curvatures



Alpha Shape in 2-dimensional space [4]

Background

How about the theory?

Alpha complex

Delaunay triangulation

Simplicial complex

k-simplex

2D/3D

Explanation will be for 2D, extending to 3D is trivial

Background

k-simplex

Definition

`k-simplex`: Any subset $T \subseteq S$ of size $|T| = k + 1$,
with $0 \leq k \leq 3(d)$ defines a `k-simplex` $\triangle_T$ that ist the convex hull of T. [8]

$k = 0$	$k = 1$	$k = 2$	$k = 3$
 vertex Δ^0	 edge Δ^1	 triangle Δ^2	 tetrahedron Δ^3

http://kurlin.org/blog/complexes-are-discretizations-of-shapes/

Background
Simplicial complex

Definition

`Simplicial complex:`

A collection C of simplices forms a `simplicial complex` if it satisfies the followig conditions:

1. for a simplex Δ_T of C, the boundary simplices of $\triangle_T$ are in C
2. for two simplices of C, their intersection is either $\emptyset$ or a simplex in C

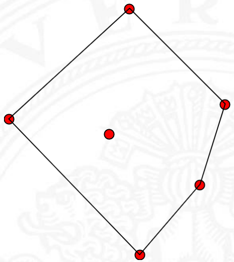
[5]

Background

Delaunay triangulation

Problem

- Given: point set S
- Underlying space: convex hull of S
- Goal: Divide $conv(S)$ into triangles with points of S as vertices.



Convex hull of a set of points

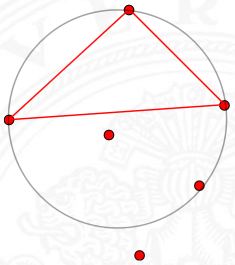
Background

Delaunay triangulation(cont.)

Algorithm

For each subset $T \subseteq S$, with $|T| = 3$

1. Test whether the circumcircle of T is `empty`
2. If yes, the points of T make up a triangle
3. otherwise discard T



Emptiness test is not successful

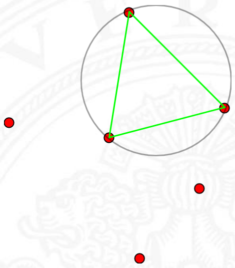
Background

Delaunay triangulation(cont.)

Algorithm

For each subset $T \subseteq S$, with $|T| = 3$

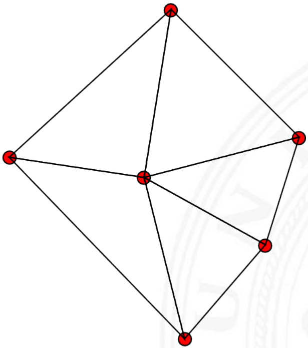
1. Test whether the circumcircle of T is `empty`
2. If yes, the points of T make up a triangle
3. otherwise discard T



Emptiness test is successful

Background

Delaunay triangulation(cont.)



Delaunay triangulation

Alpha Shapes

Alpha complex

The alpha complex C_α is a subcomplex of the Delaunay triangulation (DT)

Each k-simplex $\Delta_T \in DT(S)$ is in the alpha complex C_α if

(i) the circumcircle of T with radius $r < \alpha$ is `empty` **or**

(ii) it is a boundary simplex of a simplex of (i)

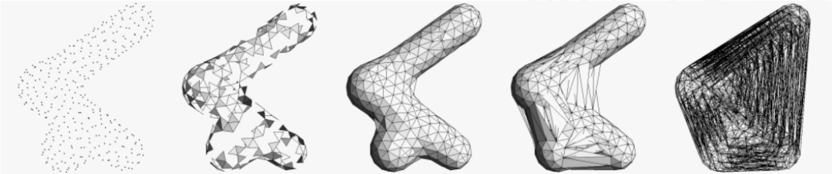
The polytope S_α then is the underlying space (i.e. union of all k-simplices Δ_T) of the alpha complex C_α:

$$|C_\alpha| = S_\alpha$$

Alpha Shapes

Family

Family of α-shapes S_α $(0 \leq \alpha \leq \infty)$



$\alpha = \{0, 0.19, 0.25, 0.75, \infty\}$ [10]

$S_0 = S$

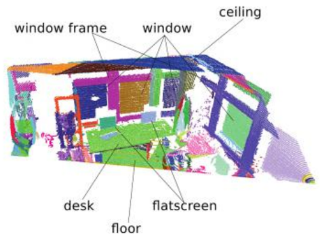
$S_\infty = conv(S)$

Application in Robotics

Scene recovery and analysis

`3D Scene Recovery and Spatial Scene Analysis for Unorganized Point Clouds` [9]

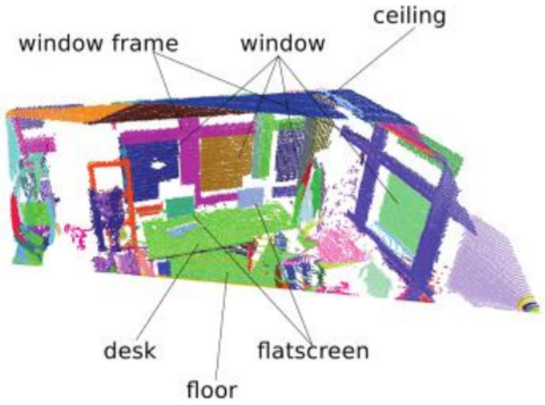
- extracting spatial entities from point clouds
- region growing as segmentation method
- surface reconstructing of each region by alpha shapes
- properties of alpha shapes are used to infer semantics



[9]

Application in Robotics

Scene recovery and analysis



Problems & Limitations

Accuracy

- Choosing the "best" α vlaue is not trivial $\rightarrow$ some (heuristical) methods
- Not for all object's surfaces there is a good α value due to non-uniformly sampled data
  - **Interstices** might be covered
  - **Neighboring** objects might be connected
  - **Joints** or sharp turns might not be sharp anymore

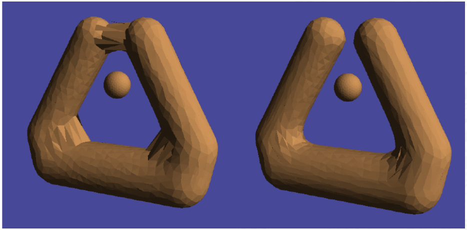
[10]

Problems & Limitations

Accuracy

Improvement: locally adjusting α test

- density scaling [10]
- anisotropic scaling [10]
- weighted alpha shapes [7]



Left: density scaling, **right**: added anisotropic scaling

Problems & Limitations

Time complexity

- Depends mostly on computation of Delaunay triangulation
- For DT in worst-case $\mathcal{O}(n^2)$, with n as number of points
- Edelsbrunner and Shar [6] developed a method for regular triangulations that performs with $\mathcal{O}(n\,log\,n)$.
  Mostly gives a complexity closer to linear. [10]

Comparison

Method	Time complexity	Robustness
Ball Pivoting [3]	linear (without DT)	Noise: **yes**; Undersampling: **no**
Voronoi Filtering [2]	quadratic (uses Voronoi Diagram)	Noise: **yes**; Undersampling: **no**
Cocone Algorithm[1]	quadratic (based on Voronoi Filtering)	Noise: **no**; Undersampling: **no**

- There are (heuristical) methods that improve robustness for each algorithm.
- Especially for undersamlpling and non-uniform sampled data by local adaption.

[1] N. Amenta, S. Choi, T. K. Dey, and N. Leekha.
A simple algorithm for homeomorphic surface reconstruction.
In *Proceedings of the Sixteenth Annual Symposium on Computational Geometry*, SCG '00, pages 213–222, New York, NY, USA, 2000. ACM.

[2] Nina Amenta, Marshall Bern, and Manolis Kamvysselis.
A new voronoi-based surface reconstruction algorithm.
In *Proceedings of the 25th Annual Conference on Computer Graphics and Interactive Techniques*, SIGGRAPH '98, pages 415–421, New York, NY, USA, 1998. ACM.

[3] Fausto Bernardini, Joshua Mittleman, Holly Rushmeier, Cláudio Silva, and Gabriel Taubin.
The ball-pivoting algorithm for surface reconstruction.
IEEE Transactions on Visualization and Computer Graphics, 5(4):349–359, October 1999.

[4] Tran Kai Frank Da, Sébastien Loriot, and Mariette Yvinec.
3d alpha shapes.
In *CGAL User and Reference Manual*. CGAL Editorial Board, 4.7 edition, 2015.

[5] International Business Machines Corporation. Research Division, B. Guo, J.P. Menon, and B. Willette.
Surface Reconstruction Using Alpha Shapes.
Research report. IBM T.J. Watson Research Center, 1997.

[6] H. Edelsbrunner and N. R. Shah.
Incremental topological flipping works for regular triangulations.
In *Proceedings of the Eighth Annual Symposium on Computational Geometry*, SCG '92, pages 43–52, New York, NY, USA, 1992. ACM.

[7] Herbert Edelsbrunner.
Weighted alpha shapes.
University of Illinois at Urbana-Champaign, Department of Computer Science, 1992.

[8] Herbert Edelsbrunner and Ernst P. Mücke.
Three-dimensional alpha shapes.
ACM Trans. Graph., 13(1):43–72, January 1994.

[9] Markus Eich and Malgorzata Goldhoorn.
3d scene recovery and spatial scene analysis for unorganized point clouds.

In *Proceedings of 13th International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines. International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines (CLAWAR-10), August 31 - September 3, Nagoya, Japan*. o.A., 8 2010.

[10] Marek Teichmann and Michael Capps.
Surface reconstruction with anisotropic density-scaled alpha shapes.
In *Proceedings of the Conference on Visualization '98*, VIS '98, pages 67–72, Los Alamitos, CA, USA, 1998. IEEE Computer Society Press.