University of Hamburg
Faculty of MIN, Department of Informatics, Group TAMS
J. Zhang, H. Bistry

Secretary: T. Tetsis
Room: F-311
Phone: 42883-2430

Introduction to Robotics (SS2013)
Assignment #2
Due: 24.04.2014, 14.00

Task 2.1: (3 points) Consider the planar robot manipulator shown in figure 1 with the joint angles θ_1, θ_2 and θ_3 constrained by the following relation: $\theta_3 = 180^\circ - \theta_1 - \theta_2$.

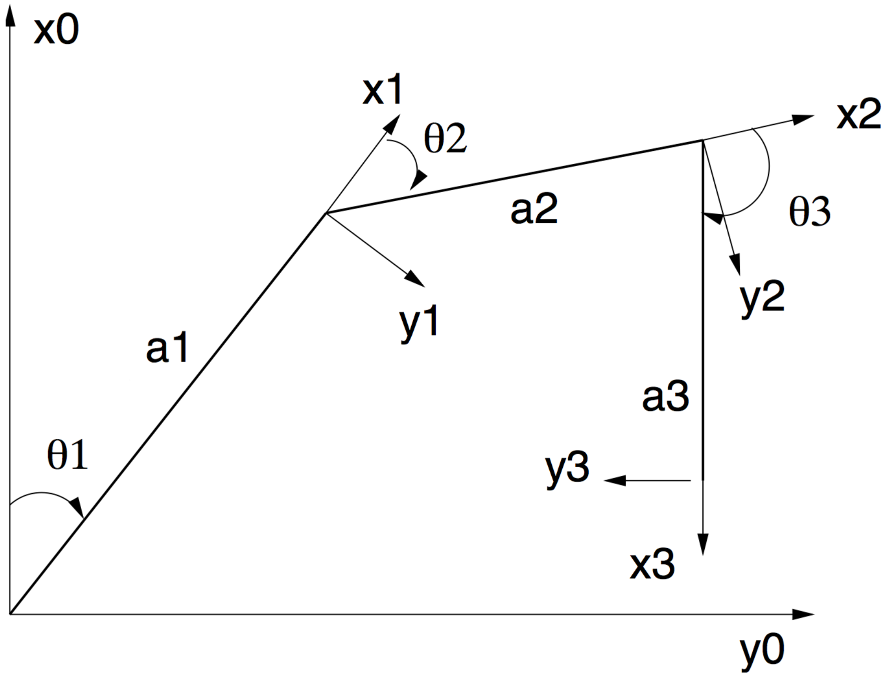

Figure 1: 3-joint planar manipulator.

2.1.1: Determine the partial homogeneous transformations ${}^{i-1}A_i,\ i = 1,2,3$ for each of the coordinate frames shown in figure 1 and show the planar manipulator transformation ${}^0T_3 = {}^0A_1\ {}^1A_2\ {}^2A_3$ to be equal to

$$
{}^0T_3 = \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -1 & 0 & S_1a_1 + S_3a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

with $C_i \equiv \cos(\theta_i)$ and $S_i \equiv \sin(\theta_i)$. **Hint:** Use the following trigonometric identities to simplify the resulting transformation matrices:

$$
\begin{aligned}
\cos(\theta_1 + \theta_2) &= \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2) \\
\sin(\theta_1 + \theta_2) &= \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2)
\end{aligned}
$$

2.1.2: Specify the two additional homogeneous transformations that are required in order to facilitate a rotation of the manipulator around the axes x_0 (angle θ_0) and x_3 (angle θ_4). It is sufficient to explicitly specify both homogeneous transformations without recalculation of the full manipulator transformation.

Task 2.2: (3 points) The Stanford manipulator has five revolute joints and one prismatic joint. Figure 2(a) illustrates the general idea and figure 2(b) shows the general configuration of a Stanford manipulator. Assume the distance between joint 1 and joint 2 to be d_2. Specify the coordinate frame of each joint and present the corresponding Denavit-Hartenberg (DH) parameters as a table.

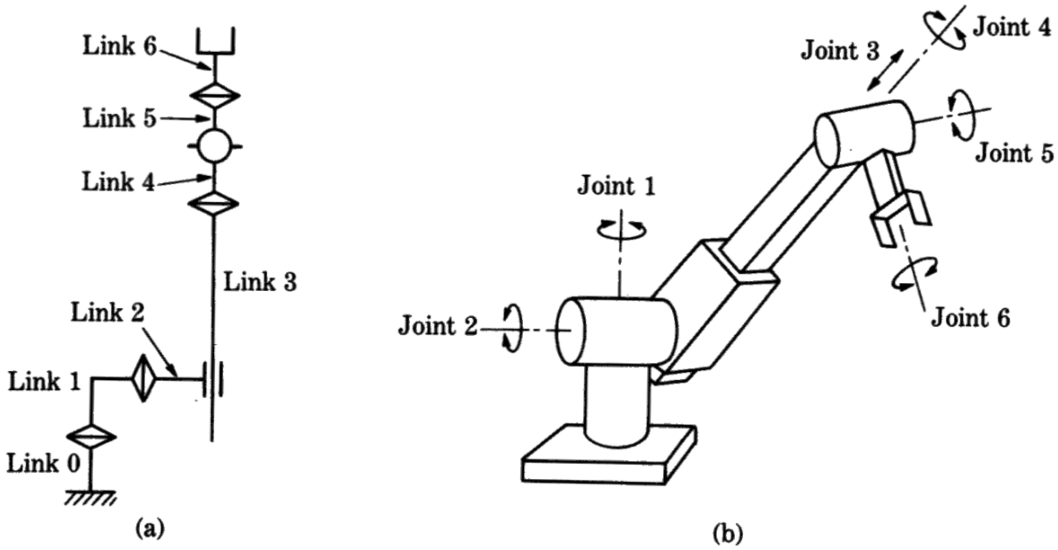

Figure 2: Stanford manipulator.

Task 2.3: (3 points) Figure 3(a) shows a 3-joint planar manipulator. Figure 3(b) shows the rotation axes of the three joints to be parallel to each other. Specify the coordinate frame of each joint and determine the corresponding DH parameters.

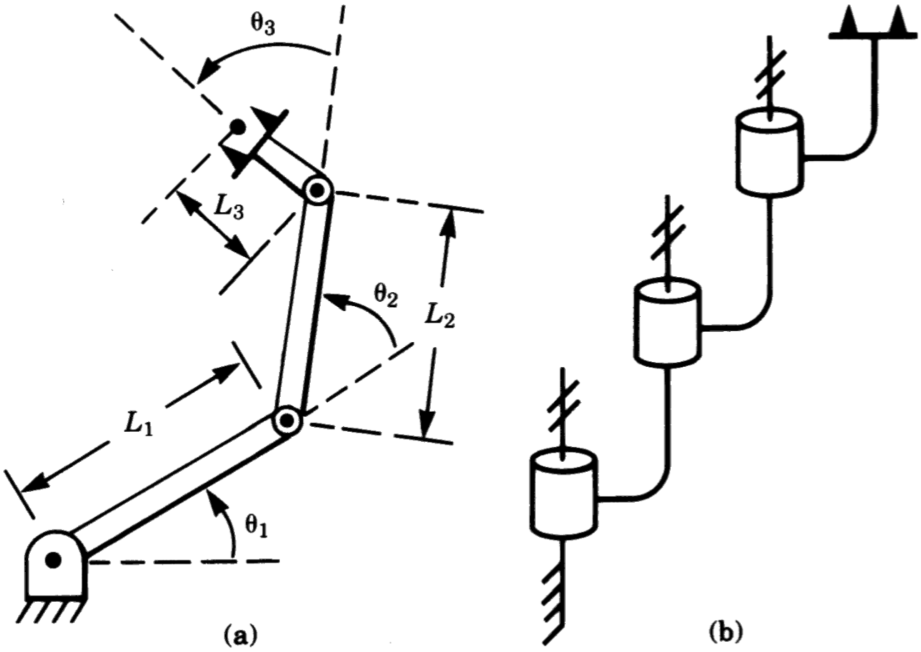

Figure 3: 3-joint planar manipulator.

Task 2.4: (3 points) An important type of manipulator is the SCARA type, a manipulator with four vertically aligned joint axes. Figure 4 shows the joint coordinate frames of such a manipulator (Adept One).

The joint variable vector is defined as $\mathbf{q} = [\theta_1, \theta_2, d_3, \theta_4]^T$. The kinematic parameters can be found in table 1:

Joint	θ	d	a	α	Zero position
1	q_1	d_1	a_1	π	0
2	q_2	0	a_2	0	0
3	0	d_3	0	0	100
4	q_4	d_4	0	0	$\pi/2$

Table 1: Kinematic parameters of the SCARA manipulator.

The "Adept One" SCARA manipulator has following values for d and a:

$$d = [877, 0, d_3, 200]^T \quad mm$$

$$a = [425, 375, 0, 0]^T \quad mm$$

2.4.1: Verify the manipulator shown in Figure 4 according to the Denavit-Hartenberg convention.

2.4.2: Determine the homogeneous transformation ${}^{Base}T_{Tool}$ of the given manipulator.

2.4.3: Determine the location of the tool center point, given the following joint value vector:

$$q = [\pi/4, -\pi/3, 120, \pi/2]^T$$

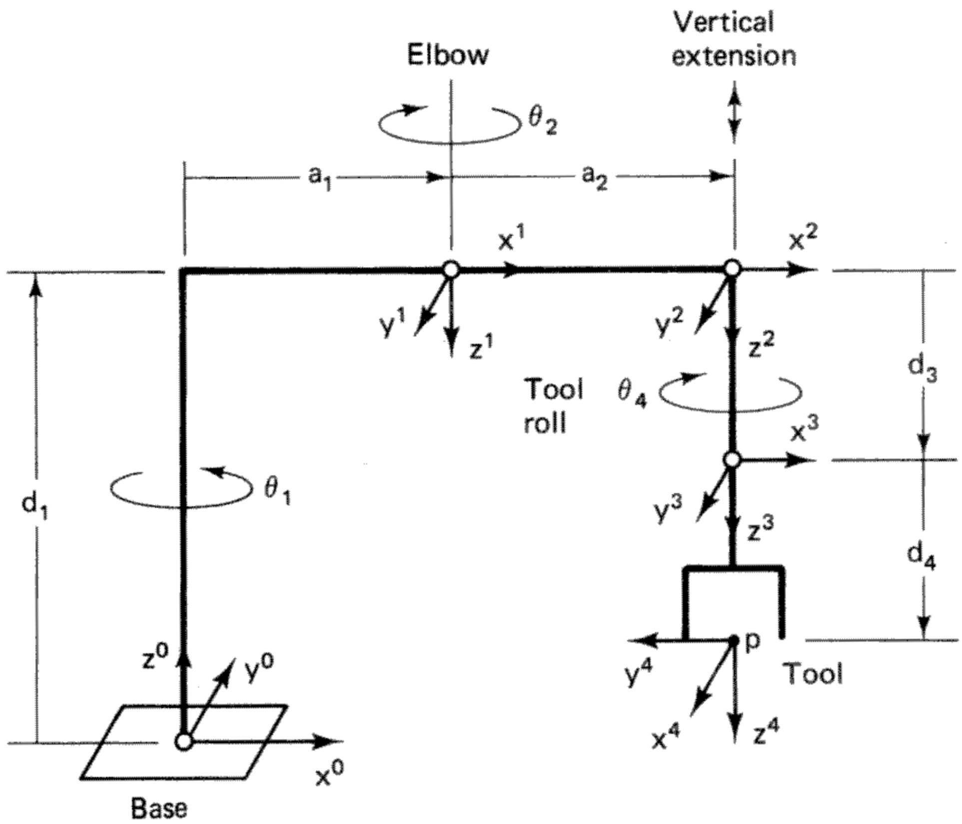

Figure 4: SCARA manipulator (Adept One).